

Spectra of turbulent flow in cumulus cloud

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HPC(NUCC ITC 2017)

Trade wind cumulus
<https://www.atmos.illinois.edu/~rauber/researchNov2005.htm>

Goal

To seamlessly simulate continuous growth of droplets
(from μm to hundreds μm radius) from the first principle

To understand the physics of cloud turbulence for more accurate prediction

- role of individual processes in the whole system
- interaction between turbulence and droplets
- validation for cloud parameterizations in terms of the first principles
- better cloud parameterizations

Equations

Turbulence

Boussinesq approximation

NS eq. + Buoyancy + modified gravity force

Temperature eq. + heat exchange due to condensation + adiab. Cooling

Water vapor mixing ratio + condensation

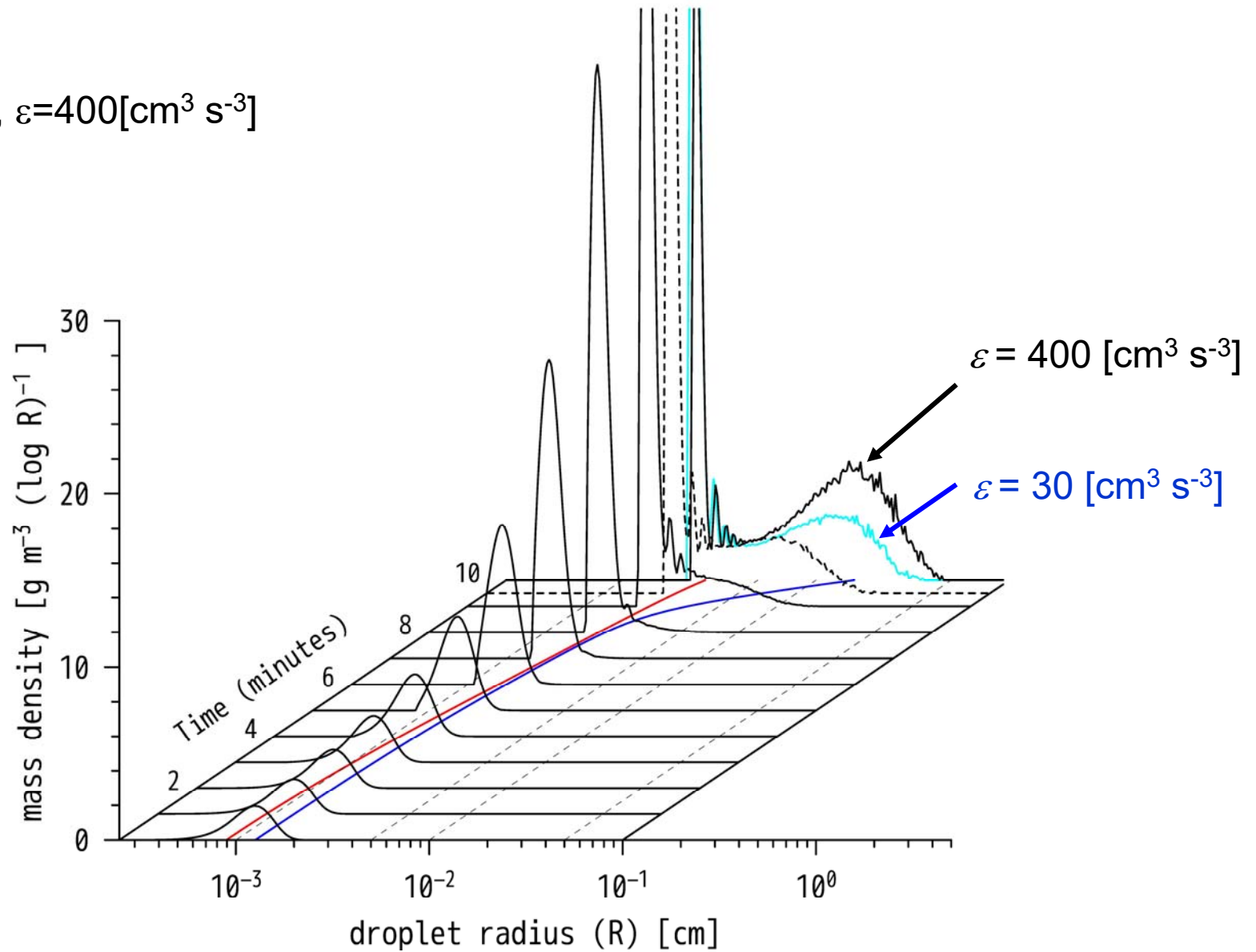
Droplets

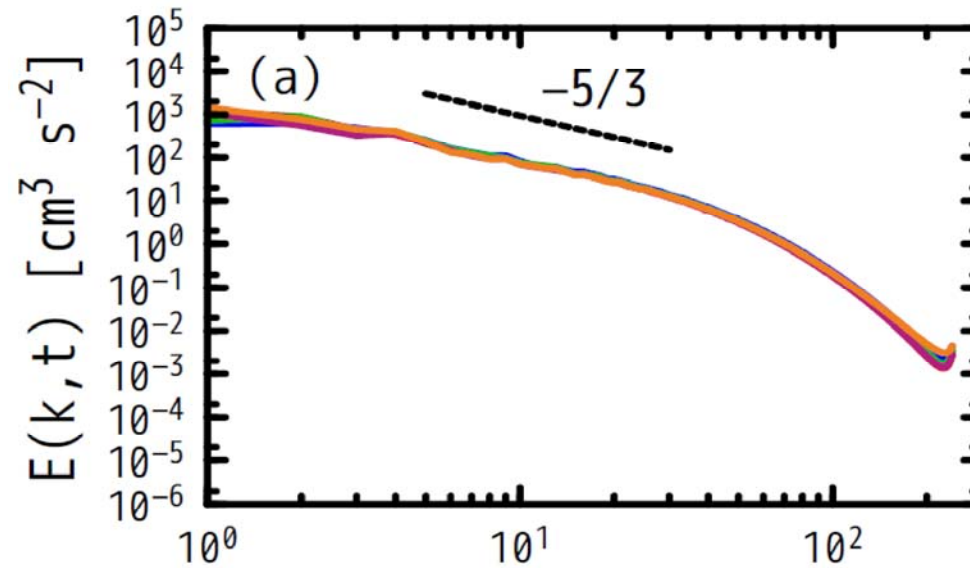
Lagrangian dynamics

Re dependent drag force + growth by cond. + collision-coalescence

Evolution of Mass Probability Density Function

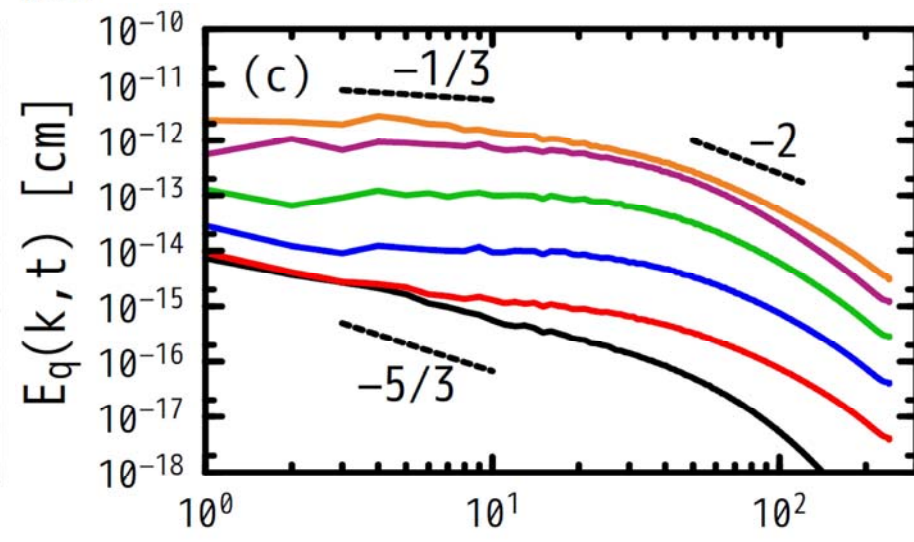
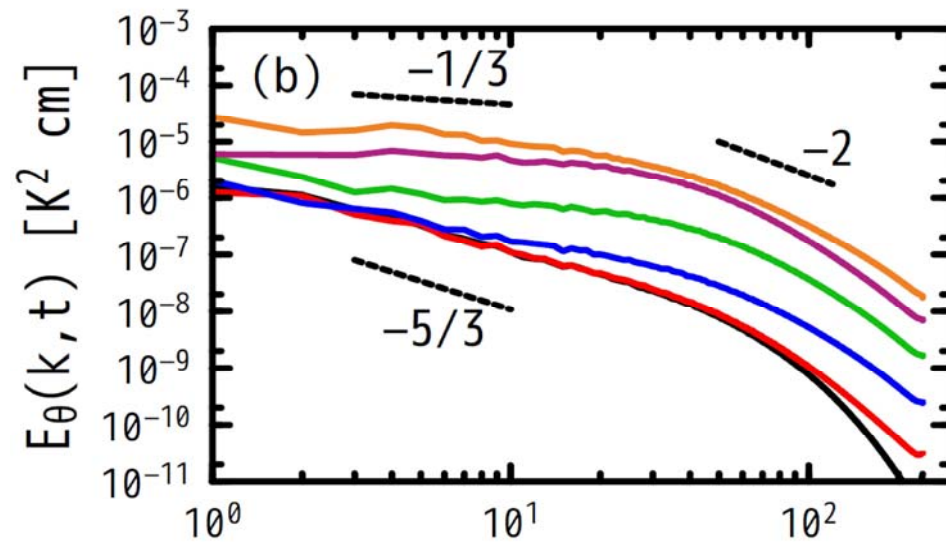
Run 4

 $R_\lambda=167$, $\varepsilon=400[\text{cm}^3 \text{s}^{-3}]$ 



Run 4

$R_\lambda = 167, \varepsilon = 400 \text{ [cm}^3 \text{ s}^{-3}\text{]}$



$t = 10, 120, 240, 360, 480, 600 \text{ [s]}$

LWC power spectrum measured at Mountain top

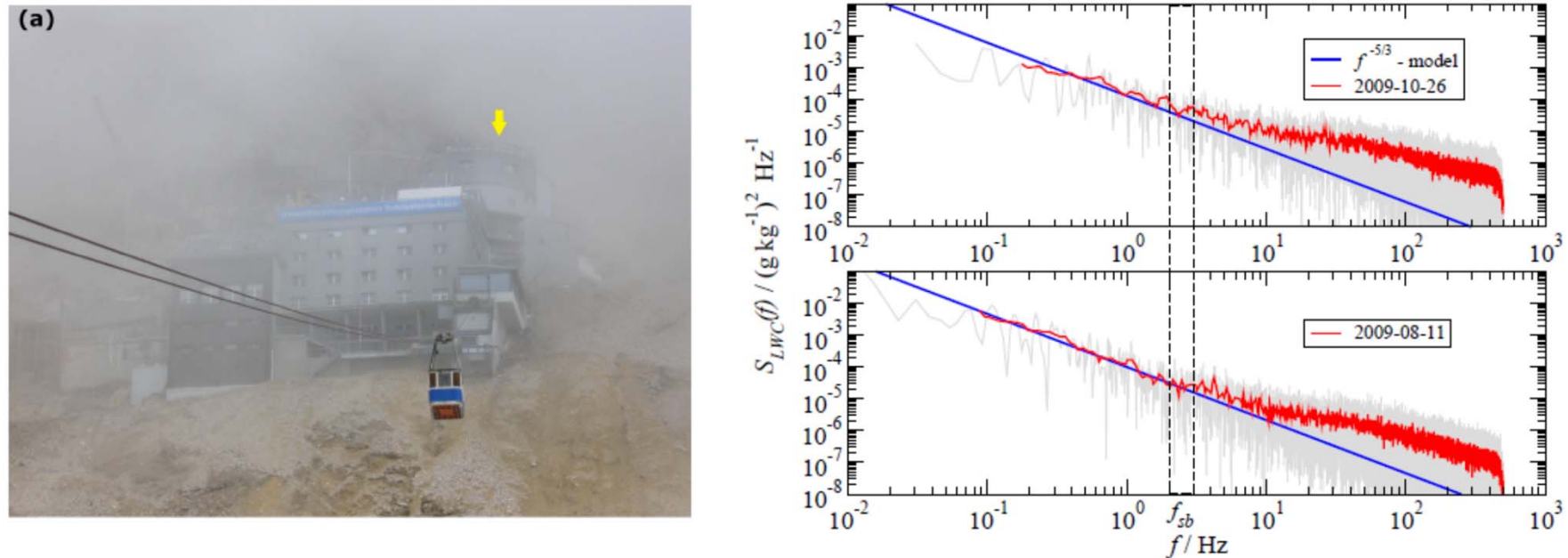


Figure 12. Power spectral densities of LWC for the 100 min long cloud period on 26 October 2009 (upper panel) and the 40 min long record on 11 August 2009 (lower panel). The light grey lines indicate the raw spectra, whereas the red lines indicate a averaged spectra. The solid blue line represents a $-5/3$ slope for inertial sub-range scaling. The vertical lines denote the scale that corresponds to the phase relaxation time estimated from the phase Doppler measurements.

H. Siebert₁, R. A. Shaw₂, J. Ditas₅, T. Schmeissner₁, S. P. Malinowski₃,
E. Bodenschatz₄, and H. Xu₄

Atmos. Meas. Tech., 8, 3219–3228, 2015

Turbulence

Boussinesq approximation

$$\frac{\partial \hat{u}}{\partial t} + \hat{u} \cdot \hat{\nabla} \hat{u} = -\hat{\nabla} \hat{p} + \nu \hat{\nabla}^2 \hat{u} + (B - \overline{B}) e_z + f, \quad \hat{\nabla} \cdot \hat{u} = 0$$

$$\frac{\partial \theta}{\partial t} + \hat{u} \cdot \hat{\nabla} \theta = \kappa \hat{\nabla}^2 \theta + \frac{L}{c_p} (C_d - \overline{C}_d) - \Gamma(t) \hat{u}_3$$

$$\frac{\partial q_v}{\partial t} + \hat{u} \cdot \hat{\nabla} q_v = \kappa \hat{\nabla}^2 q_v - (C_d - \overline{C}_d)$$

condensation, evaporation

$$\Gamma(t) = \Gamma_s(t) + \Gamma_{\text{ent}}(t)$$

$$B = g \left(\frac{T(\hat{x}, t) - T_e(H(t))}{T_e(H(t))} + 0.61(q_v(\hat{x}, t) - q_{ve}(H(t))) - q_l(\hat{x}, t) \right)$$

$$C_d(\hat{x}, t) \equiv \frac{1}{m_{\text{air}}} \frac{dm_l(\hat{x}, t)}{dt} = \frac{4\pi \rho_l K}{\rho_a (\Delta \hat{x})^3} \sum_{k=1}^{N_{\Delta}(\hat{x}, t)} R_j(t) S(\hat{X}_j + H(t) e_z, t)$$

$$q_l(\hat{x}, t) = \frac{M_l}{M_a} = \frac{4\pi \rho_l}{3\rho_a \Delta \hat{x}^3} \sum_{j=1}^{N_{\Delta}(\hat{x}, t)} R_j^3(t)$$

Two point correlation function of particle number density

$$\tilde{n} = n - \bar{n}$$

$$\langle \tilde{n}(x+r, t) \tilde{n}(x, t) \rangle = \langle n(x+r, t) n(x, t) \rangle - \bar{n}^2$$

$$= \bar{n}^2(t) w(r) + \bar{n}(t) \delta(r) - \bar{n}^2$$

$$= \bar{n}^2(t) g(r) + \bar{n}(t) \delta(r)$$

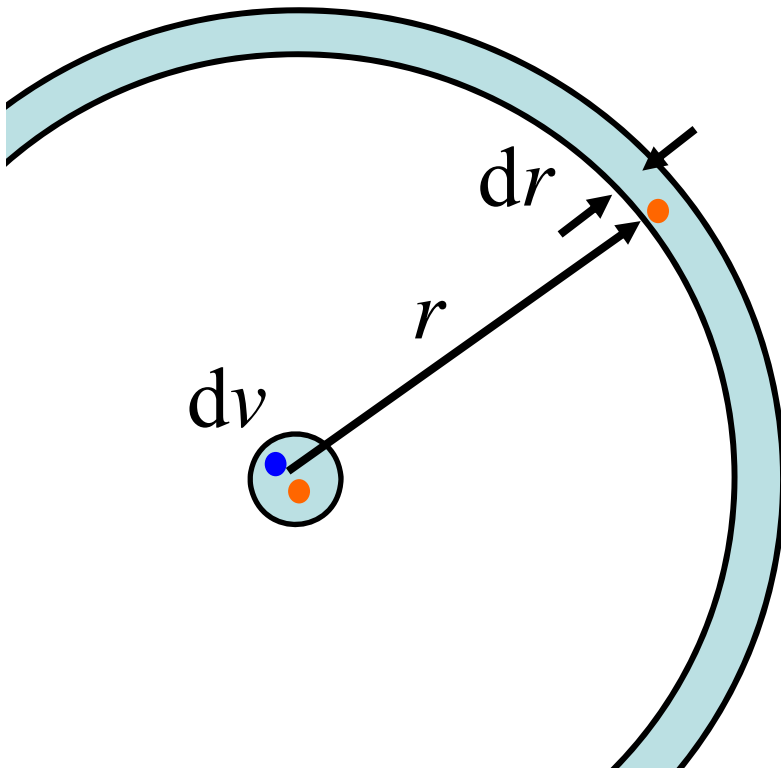
$$g(|r|) \longrightarrow 0 \quad \text{as} \quad |r| \longrightarrow \infty$$

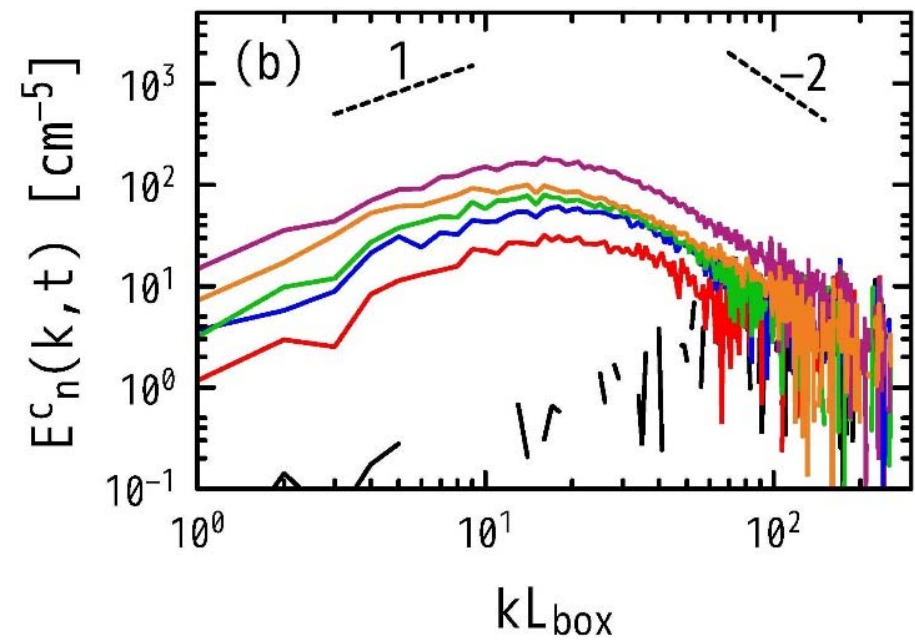
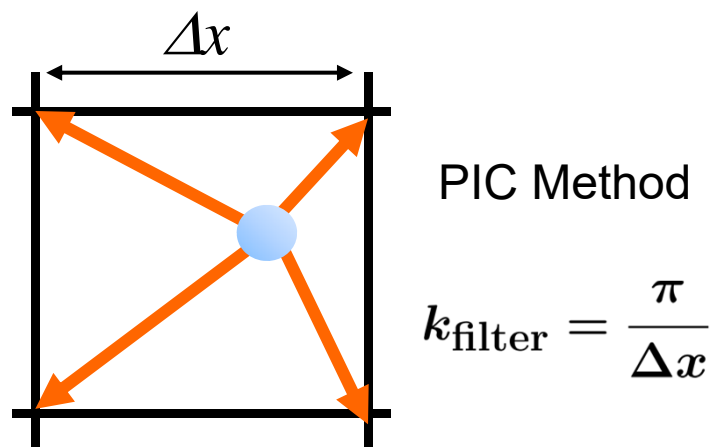
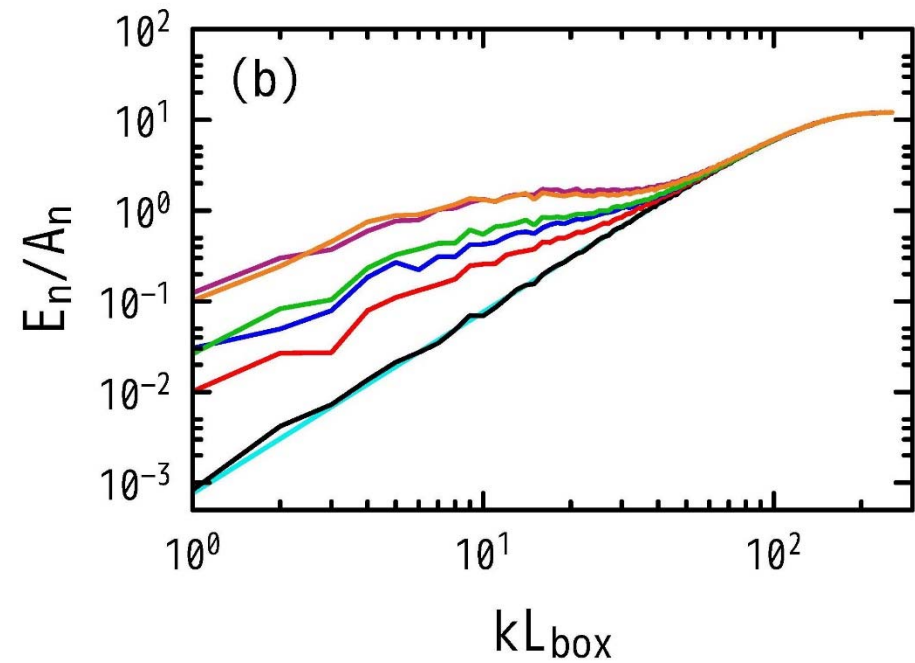
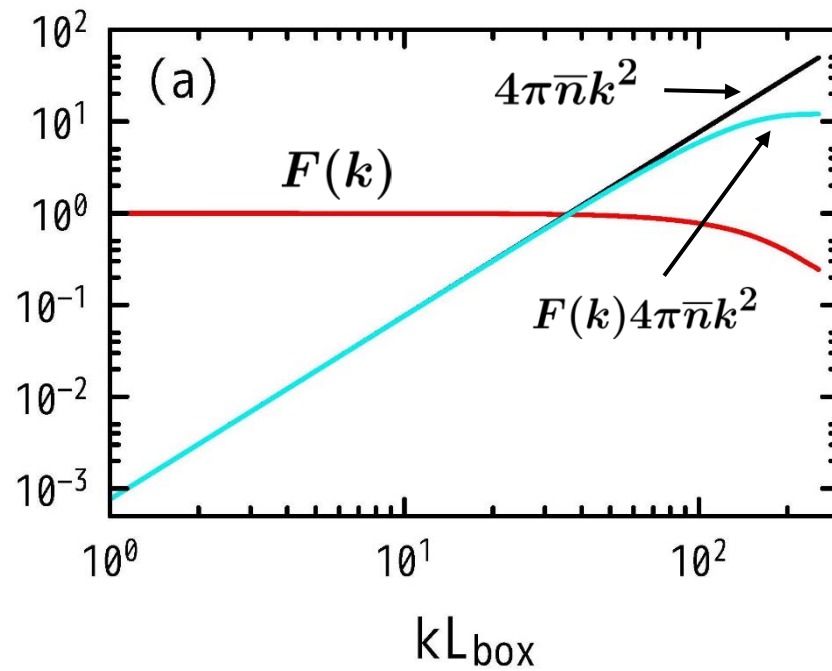
Landau Lifshitz

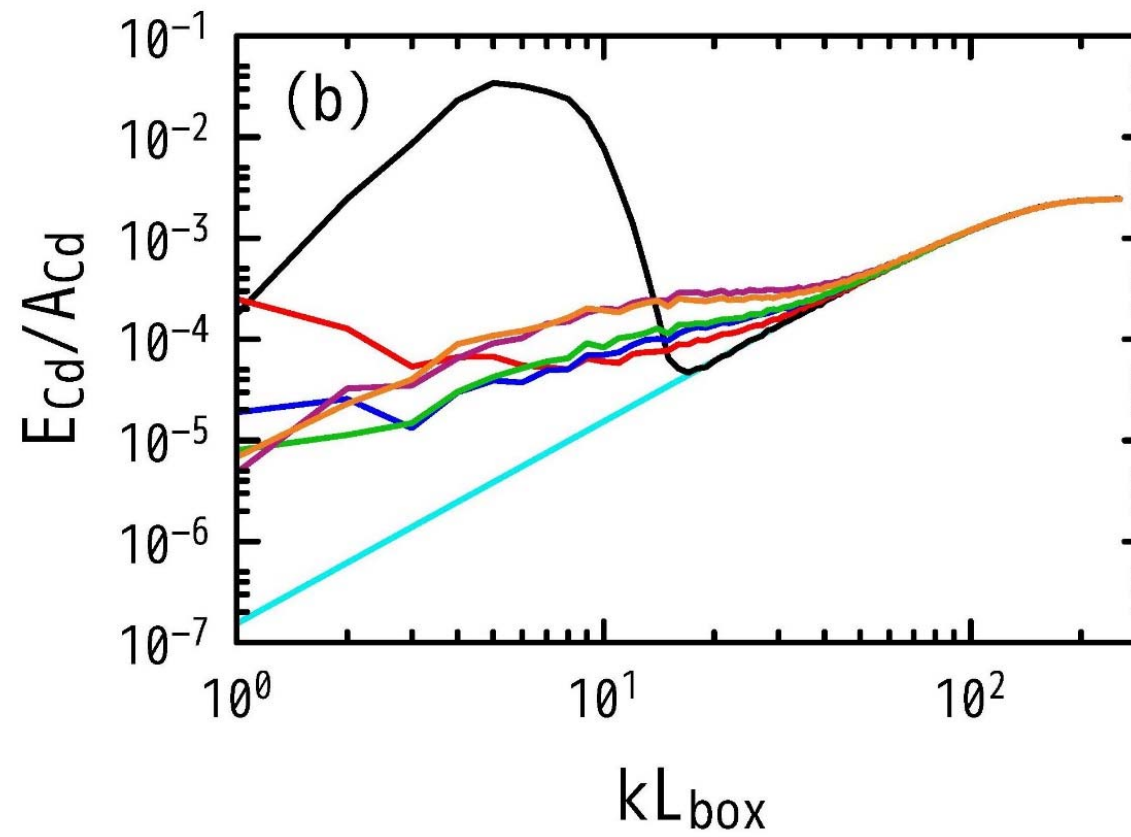
Spectrum

$$E_n(k) = E_n^c(k) + E_n^{\text{uc}}(k)$$

$$= E_n^c(k) + A_n(t) k^2$$







Equation of $E_q(k, t)$

$$\frac{\partial E_q(k, t)}{\partial t} + 2\kappa_v k^2 E_q(k, t) = T_q(k, t) + F_q(k, t),$$

$$F_q(k, t) = - \sum_{k < |k| < k + \Delta k} \left[\langle \tilde{C}_d(k, t) q(-k, t) \rangle + \langle q(k, t) \tilde{C}_d(-k, t) \rangle \right]$$

$$\begin{aligned} \sum_{k \text{ shell}} \langle \tilde{C}_d(k, t) q(-k, t) \rangle &= \sum_{k \text{ shell}} \int_0^t \langle G^L(-k, t, t') \tilde{C}_d(k, t) \tilde{C}_d(-k, t') \rangle dt' \\ &\propto \tau_q(k) E_{C_d}(k, t) \end{aligned}$$

$$E_q(k, t) \propto \tau_q(k) F_q(k, t)$$

$$\propto [\tau_q(k)]^2 E_{C_d}(k, t)$$

$$E_q(k) \propto [\tau_q(k)]^2 E_{Cd}(k)$$

Inertial range

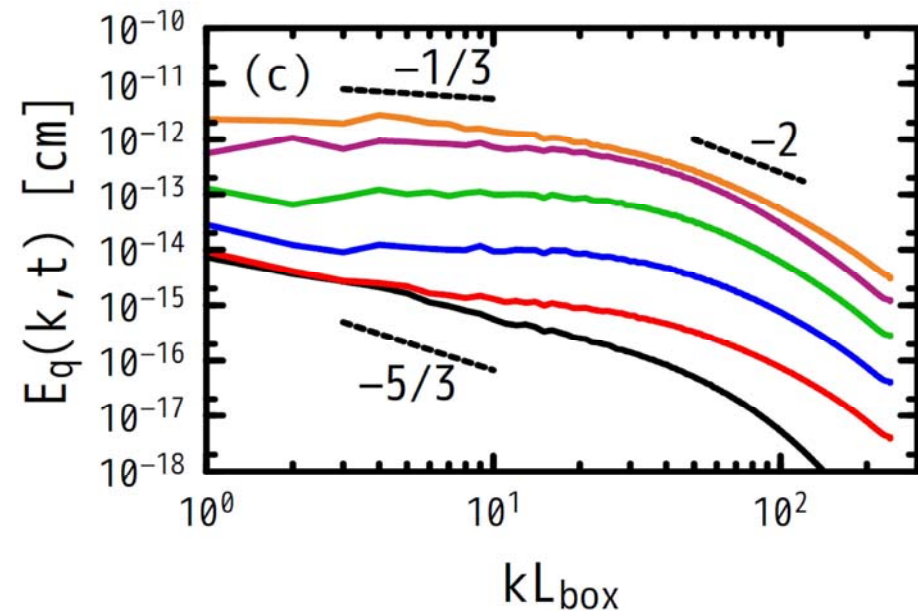
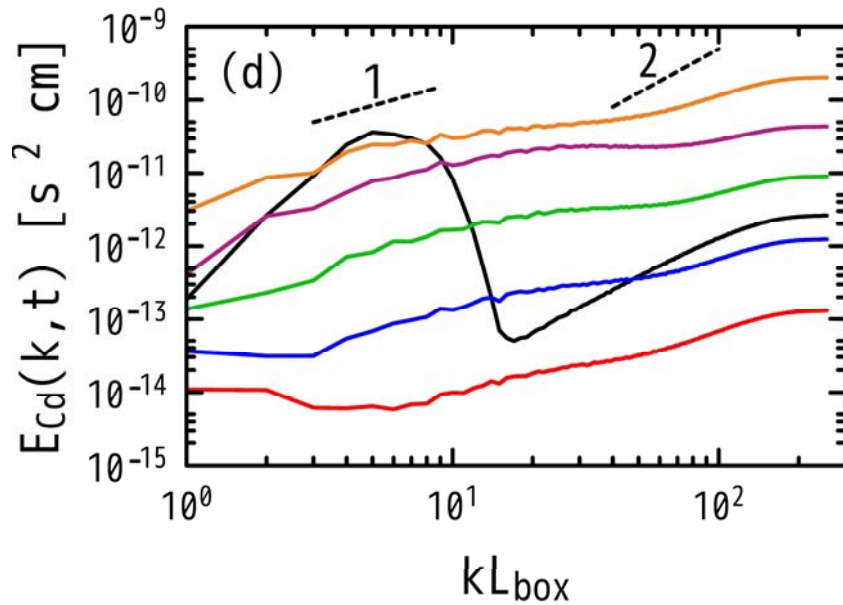
$$\tau_q(k) \propto k^{-2/3}$$

$$E_q(k) \propto k^{1-4/3} = k^{-1/3}$$

Diffusive range

$$\tau_q(k) \propto (\kappa_v k^2)^{-1}$$

$$E_q(k) \propto k^{2-4} = k^{-2}$$



Summary

- Condensation and collision/coalescence growth of cloud droplets from $\sim 10\mu\text{m}$ to $\sim 100\mu\text{m}$ was seamlessly simulated by cloud simulator.
- Spectra of turbulence are modified by the cloud droplets through the liquid water mass loading and the condensation-evaporation process
- Modification of scalar spectra extends over all wavenumbers.
- Spectra related to the cloud droplets consists of two contributions correlated part and uncorrelated part (discreteness of droplets)