

# *The Route to Raindrop Formation in a Shallow Cumulus Cloud Simulated by a Lagrangian Cloud Model*

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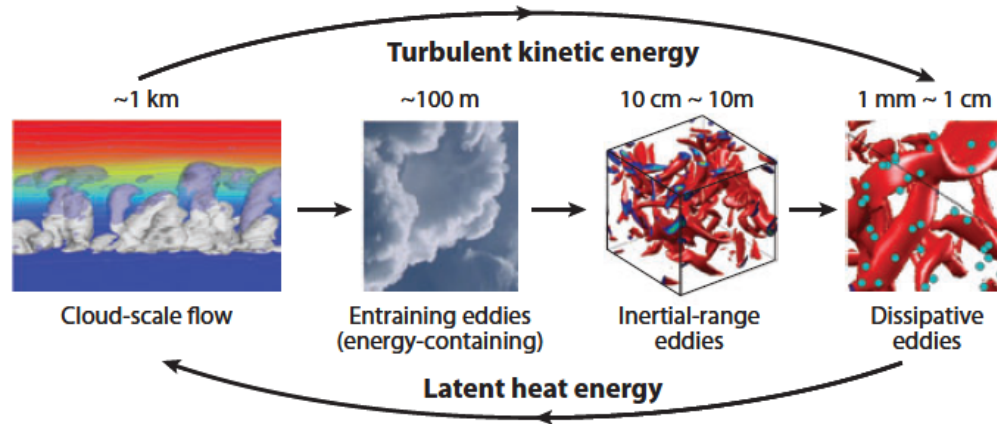
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(Hoffmann, Noh & Raasch, *JAS* 2017)

*Cloud microphysics is characterized by a particle-laden flow.*

*However, its simulation is still dominated by Eulerian models such as the spectral bin model, unlike in fluid dynamics community.*



**Figure 1**

Multiscale interactions in atmospheric clouds. The turbulent kinetic energy flows from cloud-scale motion to dissipative eddies. Latent heat energy flows from individual droplets to cloud-scale motion.

*If the Lagrangian motion of cloud droplets is calculated directly,*

- Diffusion and settling of droplets can be calculated directly.
- Activation and condensational growth are calculated naturally following droplets.
- Conversion from cloud water to rain ('autoconversion') and sweeping of cloud water by rain water ('accretion') are realized naturally.
- Time history of each droplet can be obtained.
- Direct interaction between droplets and turbulence

However, the Lagrangian LES cloud model must resolve the following problems.

***1. How to deal with an extremely large number of droplets ?***

- LWC must to be calculated by Lagrangian droplets.

***2. How to deal with droplet collision?***

- LES cannot simulate droplet collision directly unlike DNS.

## *How to deal with an extremely large number of droplets ?*

$$M_n = A_n \cdot \frac{4}{3} \pi \rho_1 r_n^3$$

$$q_l = \frac{1}{\rho_0 \Delta V} \sum_{n=1}^{N_P} M_n$$

$A_n = \textit{weighting factor}$

=  $\frac{\text{mass of real droplets per unit volume}}{\text{mass of simulated droplet per unit volume}}$

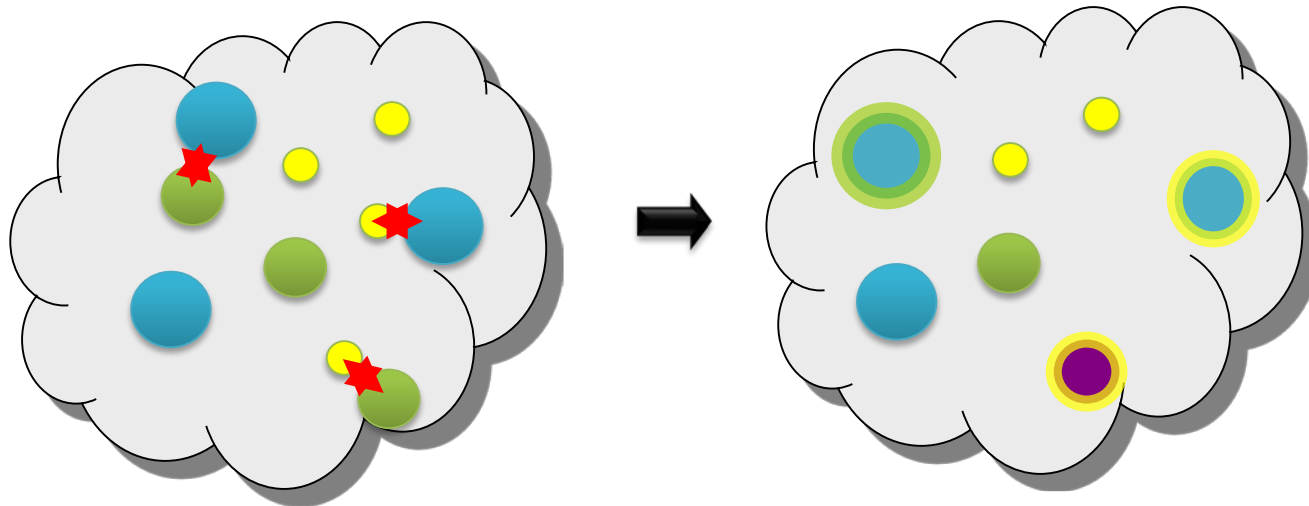
- A weighting factor ( $A_n$ ) differs for each super-droplet, and change with time as a result of collision/coalescence

# Collision/Coalescence Process

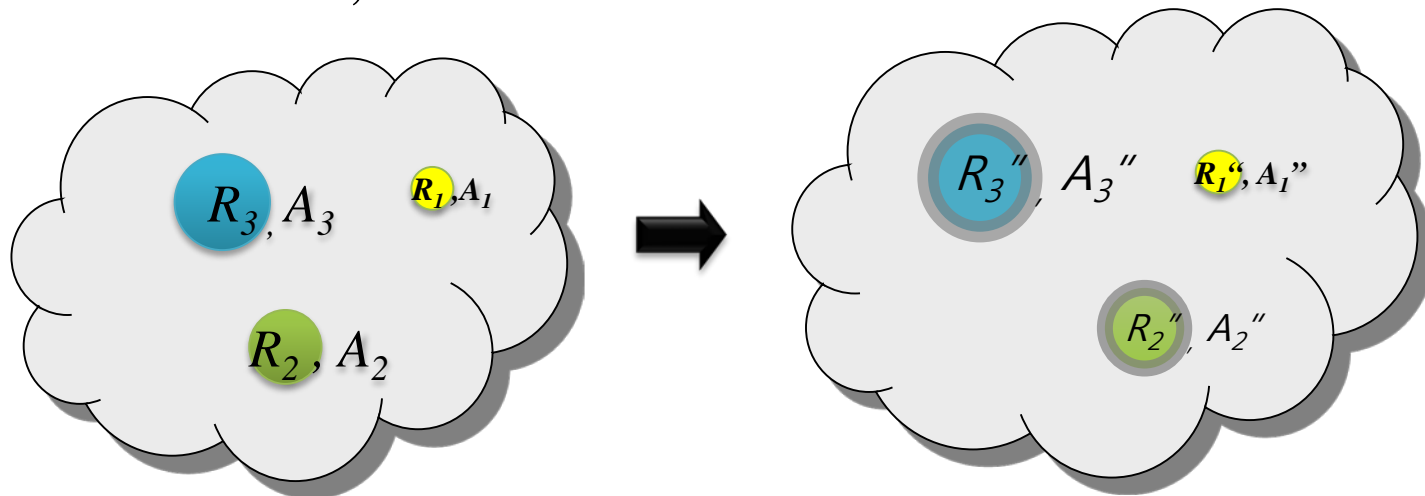
## Basic Concepts

- A statistical approach is taken in which the growth of a super-droplet is calculated based on the background droplet spectrum using the collection kernel, similar to the spectral bin model.
- Collision causes the change of the weighting factor ( $A_n$ ) and the total mass of each droplet ( $M_n$ ), which results in the change of the droplet radius ( $r_n$ ).

In the real cloud,



In the simulation,



- Particle collision is parameterized in terms of the modification of  $r_i$  and  $A_i$ .

If two super-droplets collide with  $A_m < A_n$ ,  $\rightarrow A_m, M_m(r_m)$  - increase  
 $A_n$  - decrease,  $r_n$  - invariant

$$\frac{dA_n}{dt} \delta t = -\frac{1}{2}(A_n - 1)P[K(r_n, r_n)A_n \delta t / \Delta V] - \sum_{m=n+1}^{N_P} A_m P[K(r_m, r_n)A_n \delta t / \Delta V]$$

self-collection

loss of droplets to super-droplets  
with smaller  $A_m$

$$\frac{dM_n}{dt} \delta t = -\sum_{m=1}^{n-1} A_m \frac{M_n}{A_n} P\left[\frac{K(r_n, r_m)A_n \delta t}{\Delta V}\right] + \sum_{m=n+1}^{N_P} A_n \frac{M_m}{A_m} P\left[\frac{K(r_m, r_n)A_m \delta t}{\Delta V}\right]$$

loss of mass to super-droplets  
with larger  $A_m$

gain of mass from super-droplets  
with smaller  $A_m$

$P[\varphi]$  = the probability that a collection takes place

(If  $\varphi > \xi$ , a collection takes place ( $P = 1$ ), where  $\xi$  is a random number between 0 and 1)

$\rightarrow$  **It realizes the stochastic collisional growth.**

cf. stochastic collection equation

$$\left. \frac{\partial n(r)}{\partial t} \right|_{Col} = \frac{1}{2} \int_0^r K(p, q)n(q)n(p)dpdq - n(r) \int_0^\infty K(R, r)n(R)dR$$



# Structure of Langrangian Cloud Model

(Riechelmann et al., *NJP* 2012, Lee et al., *MAP* 2014, Hoffmann et al. *AR* 2015, *JAS* 2016)

## Fluid Motion

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \dots + g \frac{\theta_v - \theta_{v0}}{\theta_{v0}} \delta_{i3}$$

$$\theta_v = \theta (1 + 0.61 q_v - q_l)$$

## SGS Turbulence

$$\frac{\partial e}{\partial t} + u_j \frac{\partial e}{\partial x_j} = \dots - \varepsilon$$

$$\varepsilon = (0.19 + 0.74l / \Delta s) e^{3/2} / l$$

## Thermodynamics

$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = - \frac{\partial}{\partial x_j} (\overline{u_j'' \theta''}) + \frac{L_e}{c_p \Pi} \Phi$$

$$\frac{\partial q_v}{\partial t} + u_j \frac{\partial q_v}{\partial x_j} = - \frac{\partial}{\partial x_j} (\overline{u_j'' q_v''}) - \Phi$$

## Microphysics

$$q_l = \frac{1}{\rho_0} \left[ \frac{\rho_L}{\Delta V} \sum_{n=1}^{N_p} A_n \frac{4}{3} \pi r_n^3 \right]$$

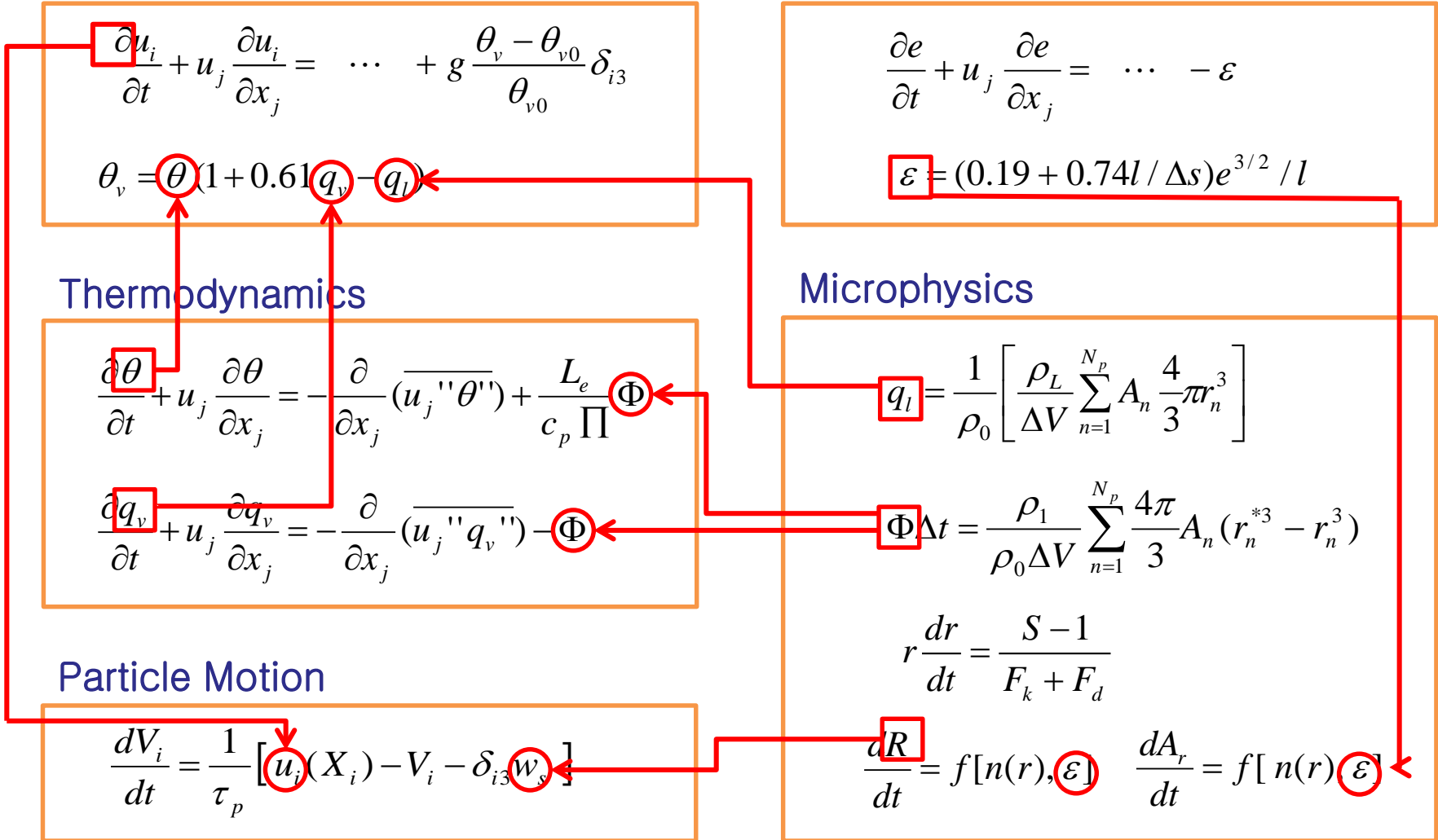
$$\Phi \Delta t = \frac{\rho_1}{\rho_0 \Delta V} \sum_{n=1}^{N_p} \frac{4\pi}{3} A_n (r_n^{*3} - r_n^3)$$

$$r \frac{dr}{dt} = \frac{S-1}{F_k + F_d}$$

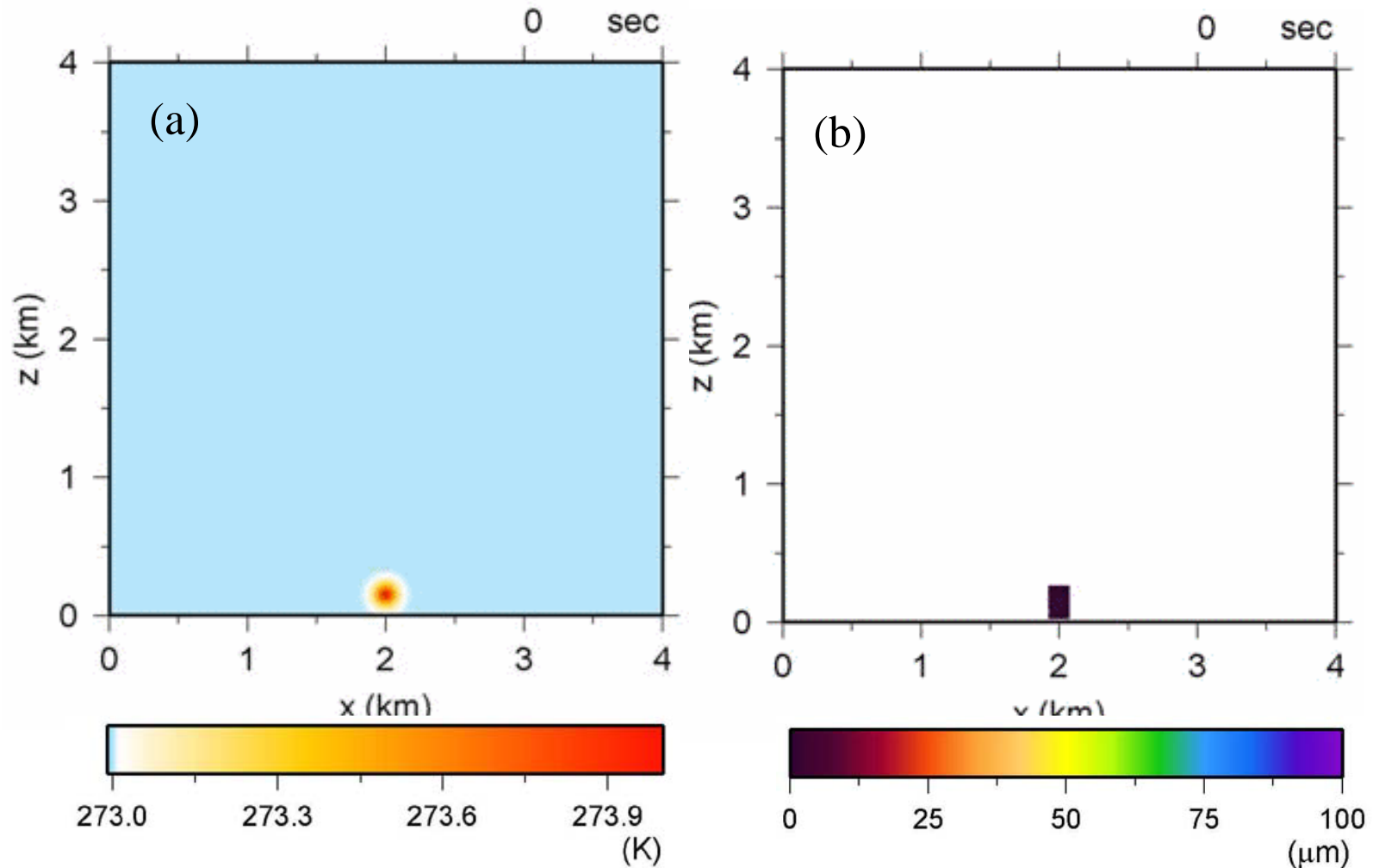
$$\frac{dR}{dt} = f[n(r), \varepsilon] \quad \frac{dA_r}{dt} = f[n(r), \varepsilon]$$

## Particle Motion

$$\frac{dV_i}{dt} = \frac{1}{\tau_p} [u_j(X_i) - V_i - \delta_{i3} w_s]$$



# Simulation of an Idealized Single Cloud



Evolutions of (a) potential temperature and (b) droplets position with radius

# Questions to Raindrop Formation

It is difficult to explain the rapid growth of cloud droplets in the size range 15 - 40  $\mu\text{m}$ , for which neither the diffusional growth and nor the collisional growth is effective.

$$\rightarrow K(R, r) = \pi(R + r)^2 |v(R) - v(r)| E(R, r)$$

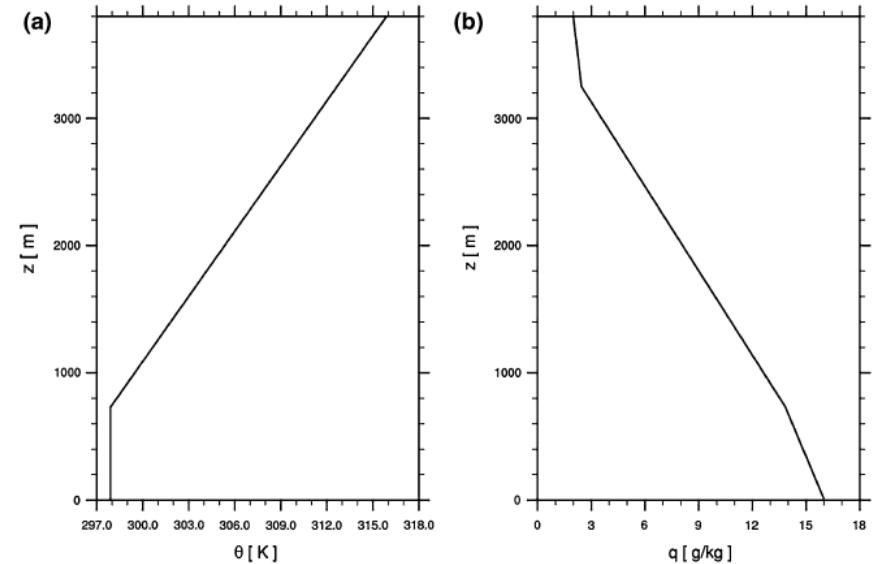
- Entrainment and mixing broaden the droplet size distribution (DSD)  
(Baker et al. 1980, Cooper 1989, Lasher-Trapp et al. 2005)
- Enhancement of the collection kernel by turbulence  
(Pinsky and Khain 2002, Wang and Grabowski 2009)
- Effects of giant aerosol particles  
(Ochs 1978, Johnson 1982)

***$\Rightarrow$  The best way to investigate raindrop formation is how and under which condition cloud droplets grow to raindrops by tracking Lagrangian droplets in LCM!***

# An Idealized Single Cloud Experiment (RICO)

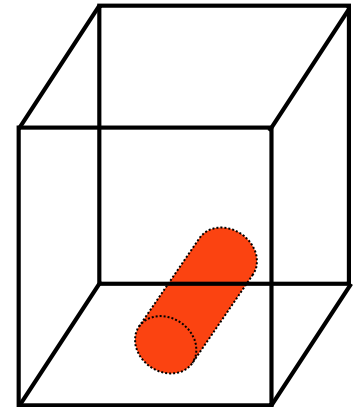
- **MODEL : LES ( PALM )**

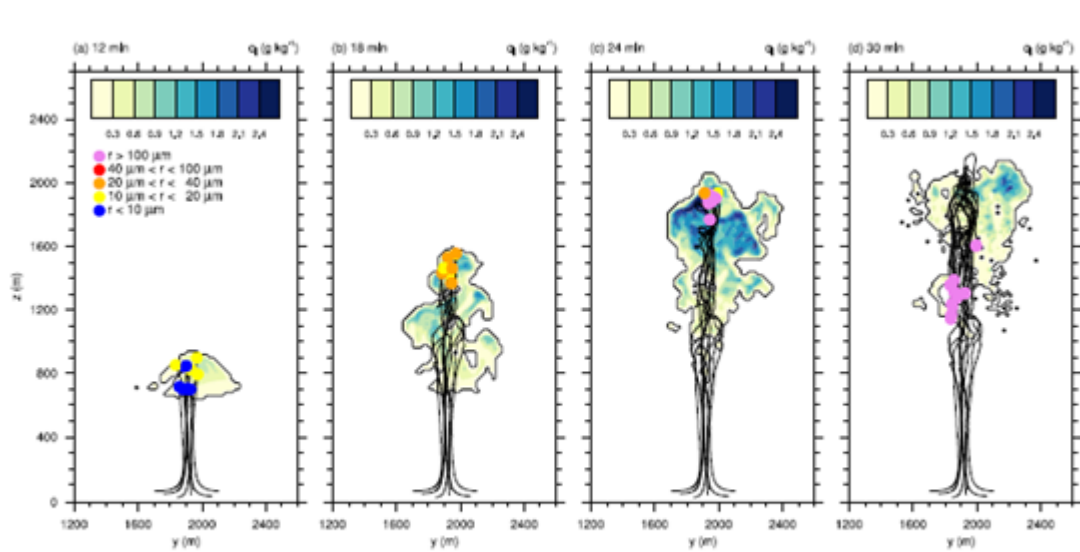
- ✓ Domain size : 1.28 km x 3.84 km x 3.84 km
- ✓ Time step: 0.1 s
- ✓ Resolution: 20 m
- ✓ Collision kernel: Hall kernel (**GRAV**)  
AW kernel (**TURB**)



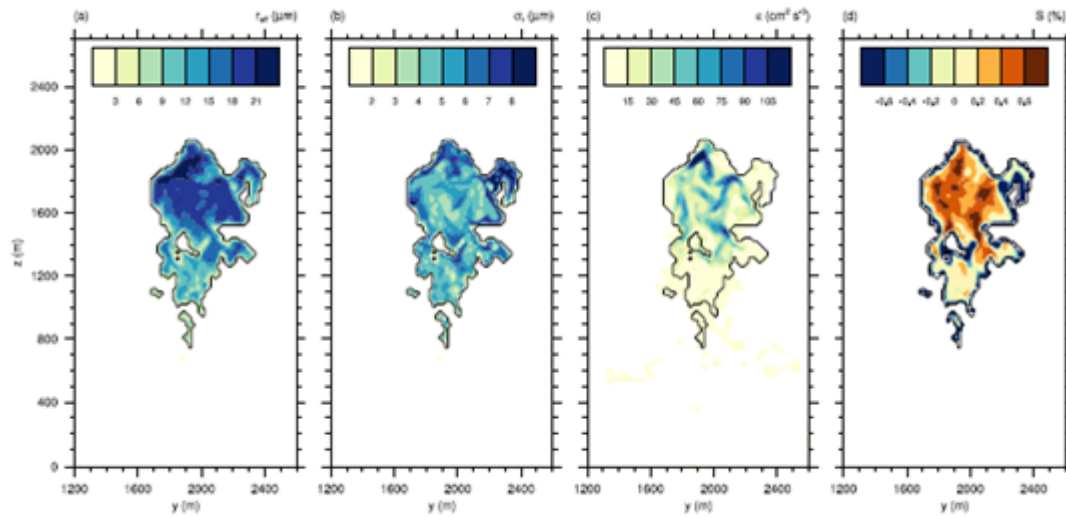
- **Particles information**

- ✓ Initial particles size: 0.1  $\mu\text{m}$
- ✓ Initial weighting factor:  $9 \cdot 10^9$
- ✓ Total number of particles:  $\sim 3.4 \times 10^8$  ( $\sim 200$  per grid box)
- ✓ Particle concentration:  $100 \text{ cm}^{-3}$
- ✓ Bubble size : 1280 m x 150 m x 200 m,  $\Delta T = 0.4 \text{ K}$
- ✓ Initial CCN concentration: **20, 70, 150  $\text{cm}^{-3}$**

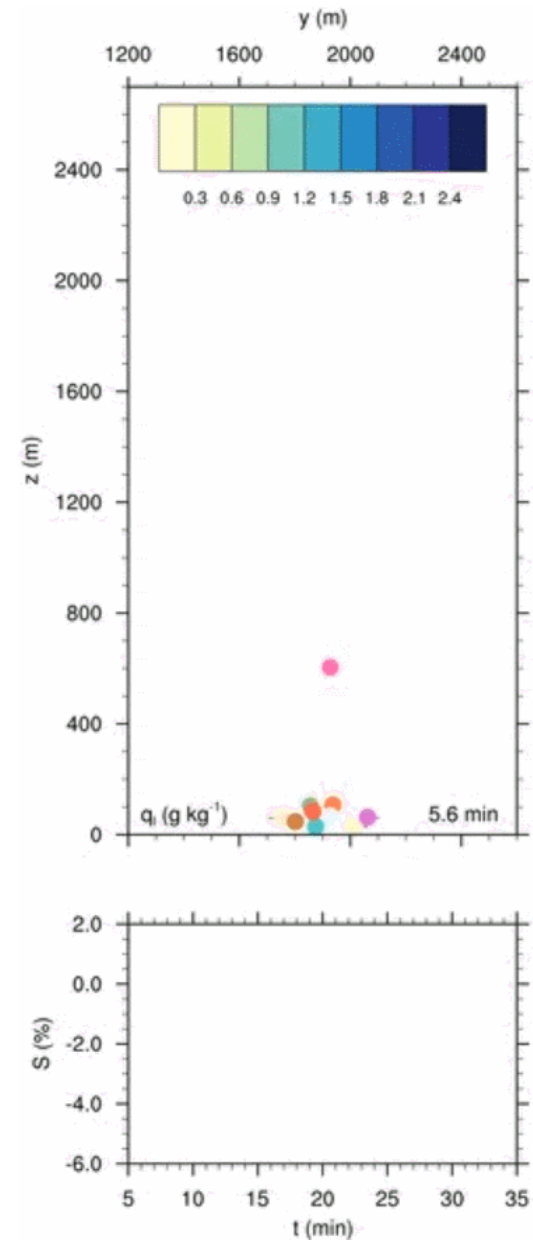




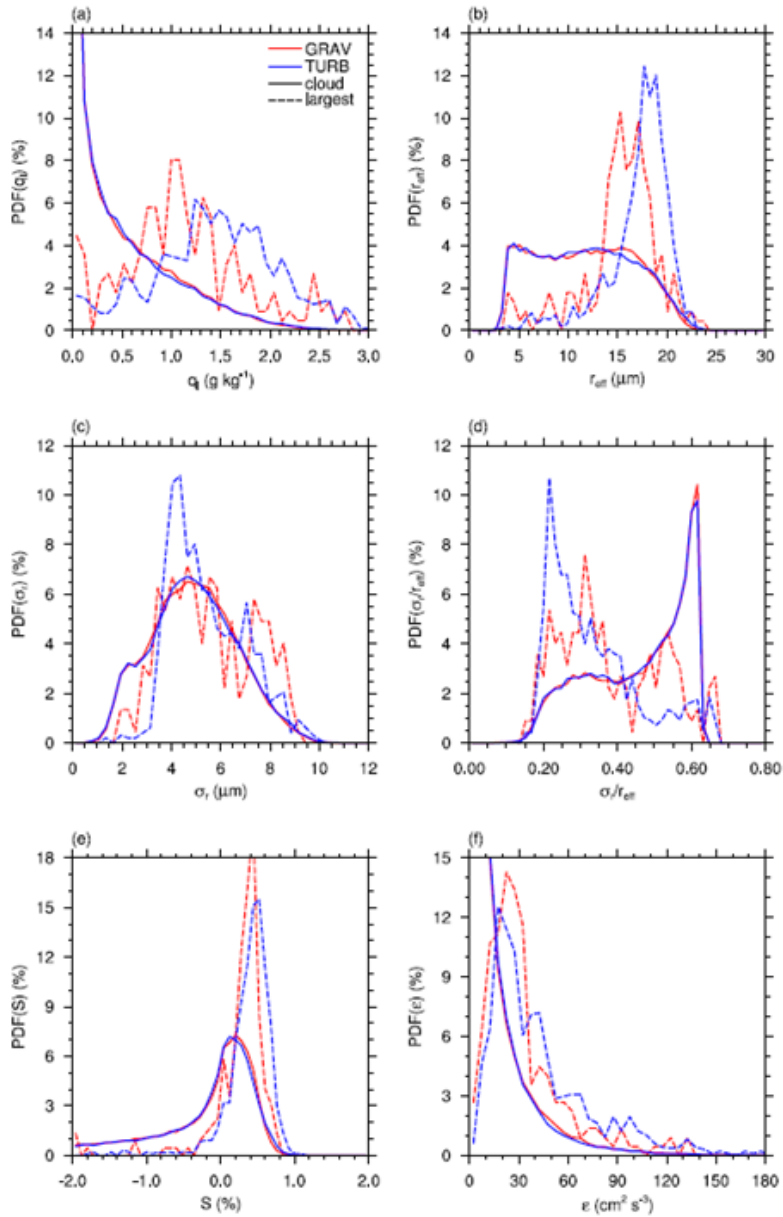
Evolution of  $q_l$  with largest droplets



Distributions of  $r_{\text{eff}}$ ,  $\sigma_r$ ,  $\epsilon$ , and  $S$



Raindrop formation is triggered when droplets with a radius of 20  $\mu\text{m}$  appear in the region near the cloud top, characterized by a large  $q_l$ ,  $\epsilon$ ,  $r_{\text{eff}}$ ,  $\sigma_r$ , and  $S$ .



## Pdf of variables

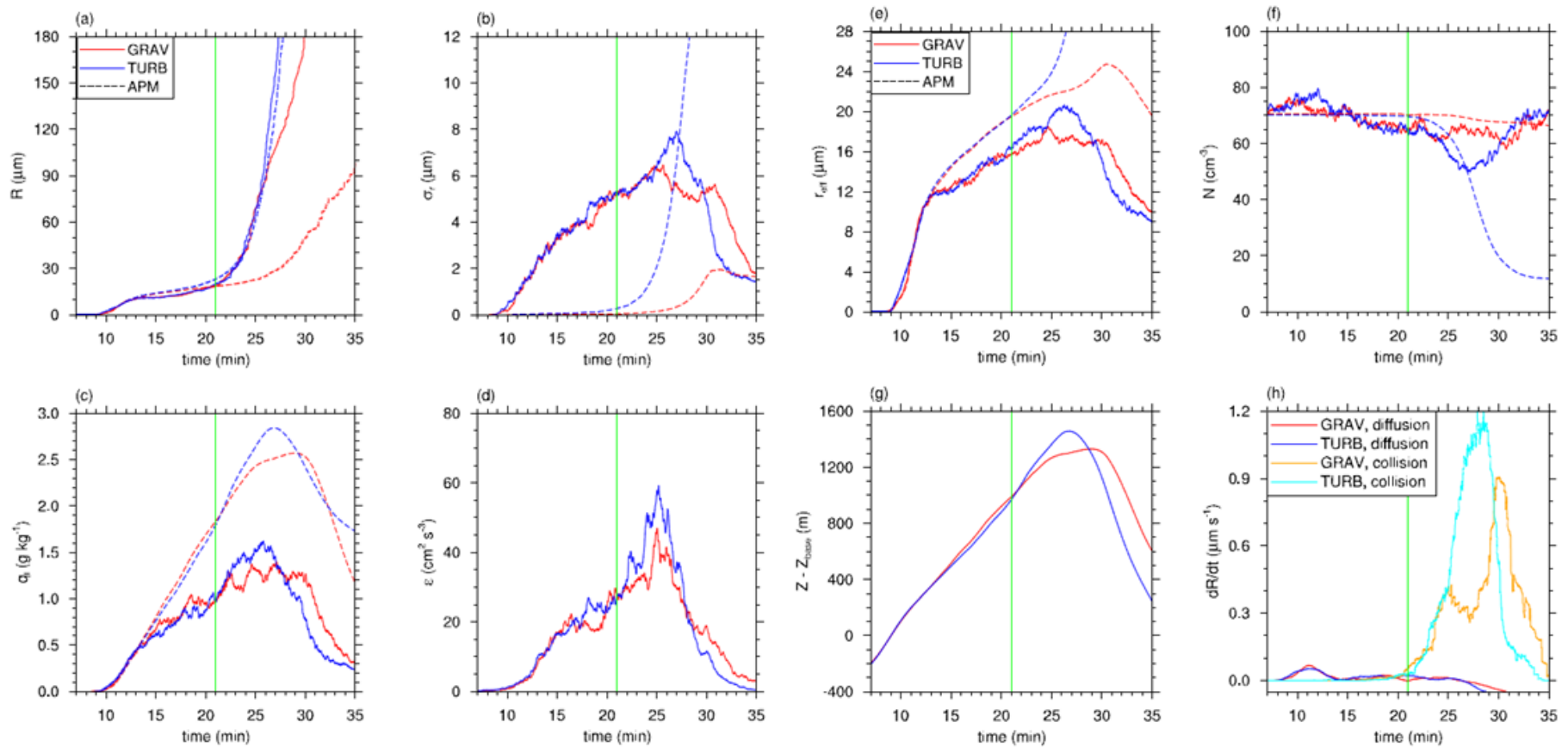
--- : potential raindrops

— : whole cloud

red: GRAV

blue: TURB

- Raindrop is formed in the region of high  $q_l$ ,  $r_{eff}$ ,  $\epsilon$ ,  $\sigma_r$  and  $S$
- TURB – higher  $q_l$ ,  $r_{eff}$ ,  $\epsilon$ , and  $S$
- GRAV – higher  $\sigma_r$



## Time series following potential raindrops

( --- : adiabatic parcel model (no DSD broadening), — : LCM; red: GRAV, blue: TURB)

- Raindrop formation is triggered, when largest droplets grow to  $r = 20 \mu\text{m}$ .
- TURB - Raindrop formation is triggered in time, regardless of DSD broadening
- GRAV – Raindrop formation is severely delayed without DSD broadening
- TURB does not accelerate the timing of raindrop formation, but it enhances the collisional growth rate substantially, leading to stronger precipitation.

# Time to reach raindrops ( $\tau_R$ ) from the collision box model

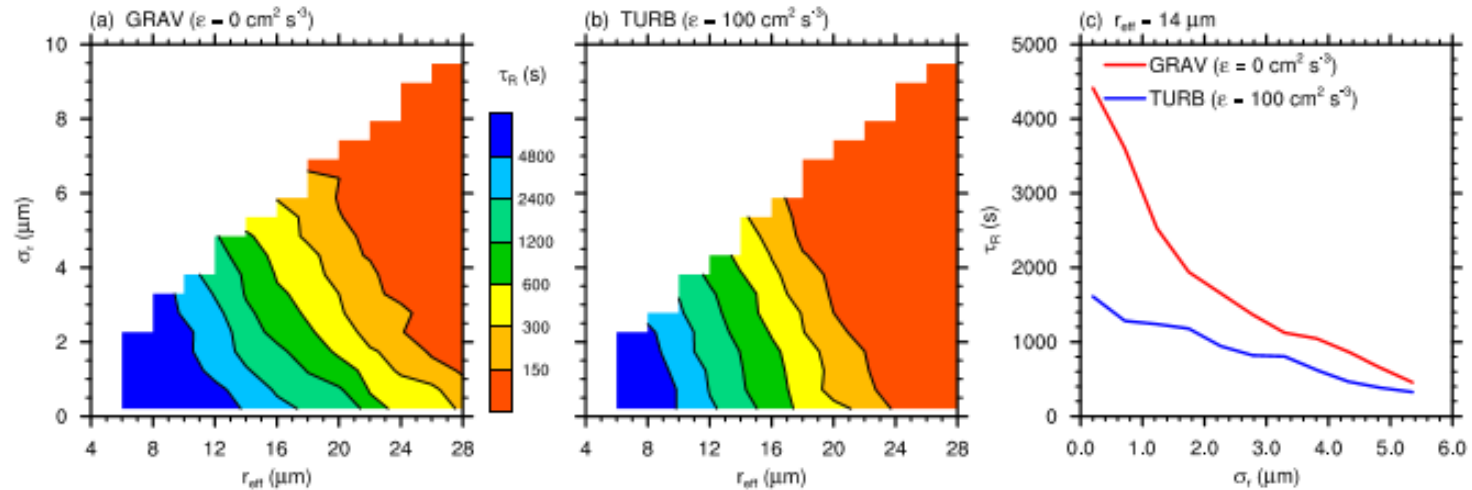
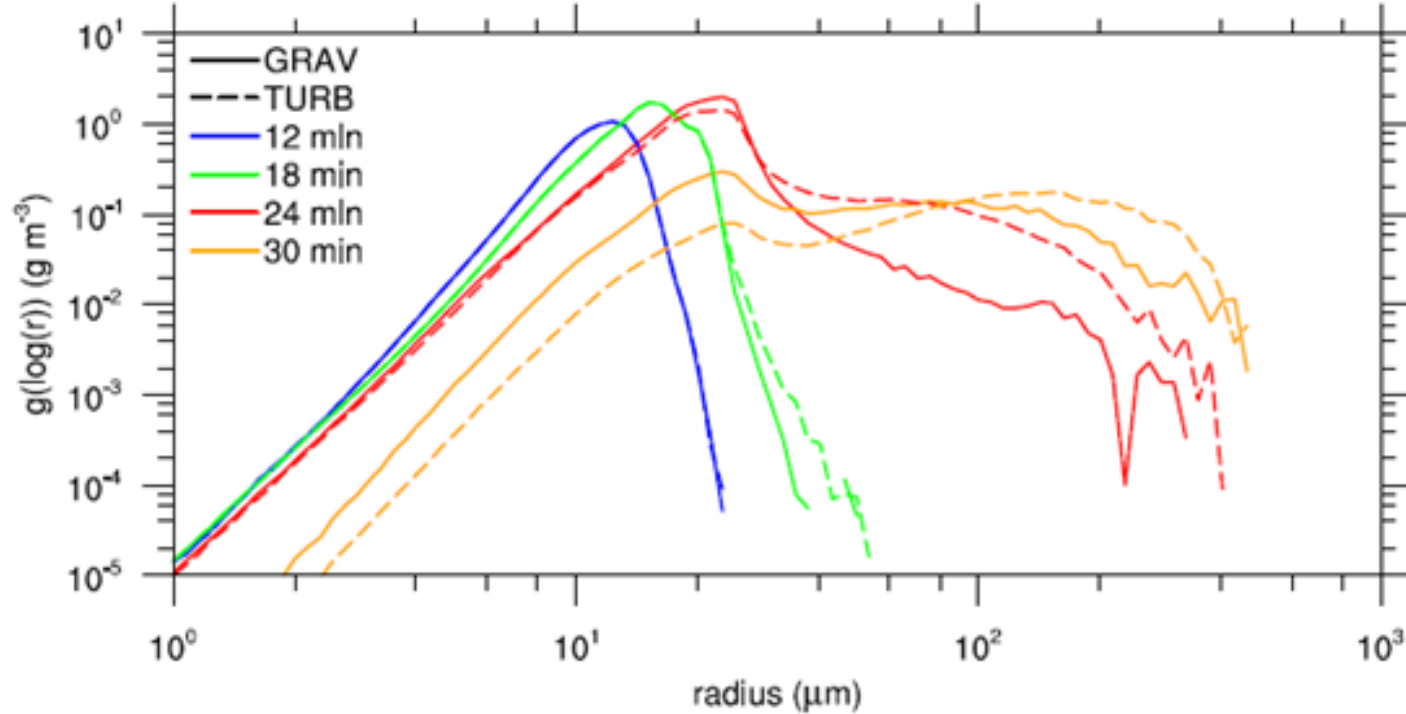


FIG. 9. The variation of the time to reach raindrops  $\tau_R$  from box simulations of the collisional growth process starting from different log-normally shaped drop size distributions with different  $\sigma_r$  and  $r_{\text{eff}}$ : (a) GRAV ( $\varepsilon = 0 \text{ cm}^2 \text{ s}^{-3}$ ), (b) TURB ( $\varepsilon = 100 \text{ cm}^2 \text{ s}^{-3}$ ), and (c) the variation of  $\tau_R$  with  $\sigma_r$  for  $r_{\text{eff}} = 14 \mu\text{m}$ .

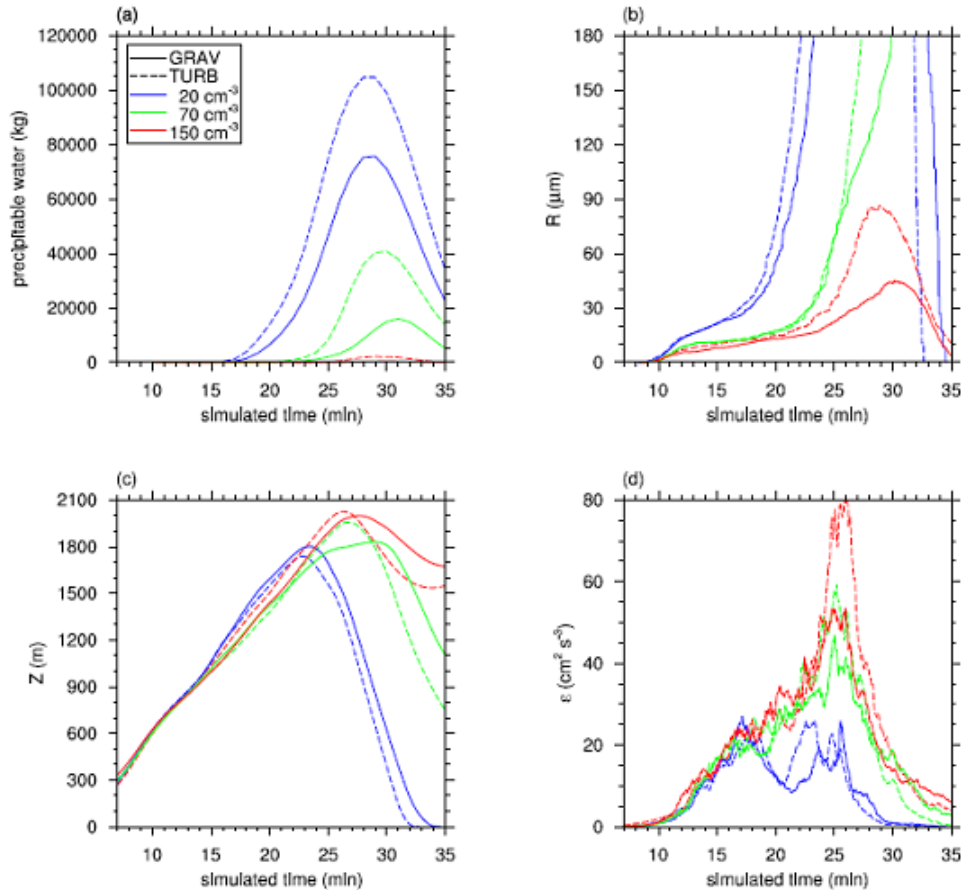
- small  $\varepsilon \rightarrow \tau_R$  becomes very large for small  $\sigma$ .
- large  $\varepsilon \rightarrow \tau_R$  does not vary much with  $\sigma$ .





Evolution of droplet spectrum

- $q_l$  ( $r > 40$   $\mu\text{m}$ ) appears at the same time, but larger at TURB
- $q_l$  ( $r < 40$   $\mu\text{m}$ ) decreases at 30 min by the collection to settling raindrops (accretion)



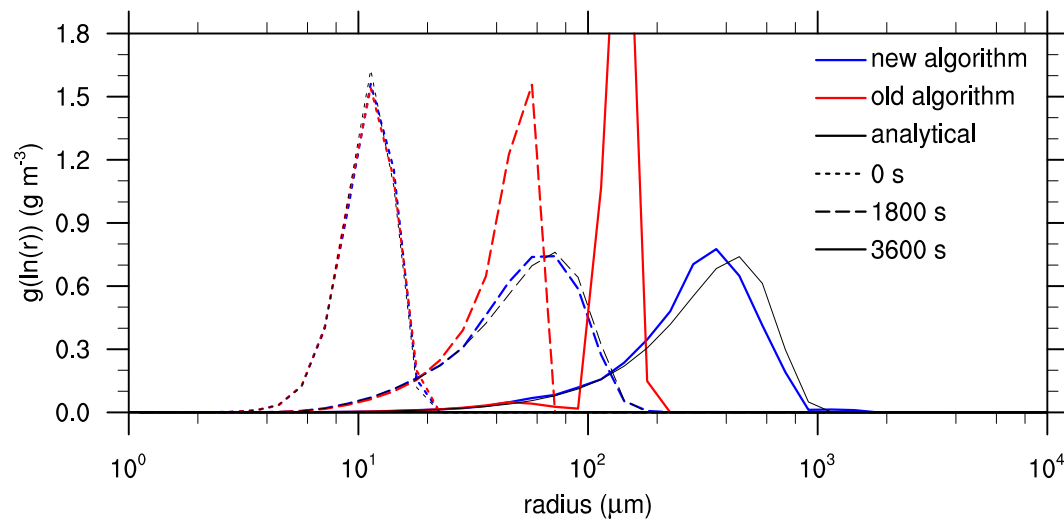
Times series of variables from different initial droplet concentrations ( $N_c$ ): (a)  $q_l$  ( $r > 40 \mu\text{m}$ ), (b)  $R$ , (c)  $Z$ , and (d)  $\varepsilon$  (solid: GRAV, dotted: TURB) (blue: 20  $\text{cm}^{-3}$ , green: 70  $\text{cm}^{-3}$ , red: 150  $\text{cm}^{-3}$ ).

- Delayed raindrop formation for larger  $N_c$ .
- stronger effect of TICE for larger  $N_c$ .

# Conclusion

- Raindrop formation is triggered
  - when droplets with a radius of  $20\ \mu\text{m}$  appear in the region near the cloud top
- Raindrop is formed in the region of high  $q_l$ ,  $r_{\text{eff}}$ ,  $\varepsilon$ ,  $\sigma_r$  and  $S$ .
  - $q_l$ ,  $r_{\text{eff}}$ ,  $\varepsilon$ , and  $S$  are higher in TURB
  - $\sigma_r$  is higher in GRAV.
- TURB - Raindrop formation is triggered in time, regardless of DSD broadening.  
GRAV – Raindrop formation is severely delayed without DSD broadening.
- TURB does not accelerate the timing of raindrop formation, but it enhances the collisional growth rate substantially, leading to stronger precipitation.
- As aerosol concentration ( $N$ ) increases,
  - faster and stronger precipitation
  - stronger effect of turbulence

<b>old algorithm</b> (continuous growth)	<b>new algorithm</b> (stochastic growth)
The super-droplet with the larger radius collected droplets from the super-droplet with the smaller radius.	The super-droplet with the smaller weighting factor collects droplets from the super-droplet with the larger weighting factor.
The collected mass is distributed among a much larger number of droplets represented by the collecting super-droplet	The collection is treated as a zero-one process, in which either all droplets of the collecting super-droplet coalesce with the same number of droplets from the collected super-droplet or not.



Evolution of droplet spectrum by collision  
(Unterstrasser et al., *GMD* 2016)