FRACTAL SUB-GRID SCALE MODEL FOR LARGE EDDY SIMULATION OF ATMOSPHERIC TURBULENCE: 2D *A PRIORI* TEST

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ABSTRACT

We implement a fractal sub-grid scale model of Scotti & Meneveau (1999) for large eddy simulation (LES) of atmospheric flow. The fractal model is based on the fractality assumption of turbulent velocity field. It reconstructs subgrid velocity field from the knowledge of its LES velocity field, using fractal interpolation technique (FIT). The characteristics of the reconstructed signal depends on the (free) stretching parameters d, which is related to the fractal dimension of the signal (which is known or assumed a priori).

To improve this method and account for the stretching parameter variability, we use the geometrical method proposed by Mazel & Hayes (1992) to calculate the probability distribution function (pdf) of the stretching parameter d in Direct Numerical Simulation (DNS) data of stratocumulus-top boundary layer (STBL) (courtesy of Prof. J.-P. Mellado). We observe self-similarity in the pdfs of d when the velocity fields are filtered to wave-numbers within the inertial range. Also, we compare the pdf of d for DNS, LES data of stratocumulus-top boundary layer and Physics of Stratocumulus Top (POST) measurements (Jen-La Plante *et al.*, 2016). By randomly selecting d from its pdf, we perform a 2D a priori test and compare statistics of the constructed velocity increments with DNS velocity increments.

INTRODUCTION

Complex spatial and temporal structures are the major characteristics of atmospheric flows. These structures are seen over a wide range of scales from large synoptic scales $\mathcal{O}(1000km)$ to the smallest dissipative scales $\mathcal{O}(1cm-1mm)$ with Reynold number $L/\eta \approx \mathcal{O}(10^9)$. All scales play important role in weather prediction including the small scales, which influence cloud droplet clustering and affect average settling velocity of droplets with Stokes number much smaller than 1. Direct numerical simulation is an ideal approach to resolve all these scales but it imposes an unrealistic computational cost. Alternatively, large-eddy simulation (LES) allows for significantly improved accuracy in simulating atmospheric (turbulent) flows, by calculating the large scale features of the flow while the interactions between large (resolved) scales and small (unresolved) scales are accounted for by a subgrid-scale model. Structural sub-grid models such as FIT was introduced to construct synthetic, fractal subgrid-scale fields applied to large eddy simulation of both steady and freely decaying isotropic turbulence (Scotti & Meneveau, 1999). This model was aimed at mimicking (some of) the subgrid scales, by making an approximate reconstruction of two-point particle statistics at the subgrid scales. In Scotti & Meneveau (1999), it was assumed that the stretching parameters d are constant in space and time in homogeneous and isotropic turbulence. These parameters were set to $d = \pm 2^{-1/3}$. One of the main set back of FIT is the estimate of the stretching parameters in non-homogeneous flows (Marchioli, 2017) where the use of constant stretching parameter in space and time will be unrealistic. The aim of this work is to develop an improvement to the fractal interpolation technique (FIT), which can be used as a closure model for Lagrangian tracking of particles in atmospheric turbulence. To account for the variability of the stretching parameter, we compute the local stretching parameter with a method proposed by Mazel & Hayes (1992) and use its pdf for the fractal interpolation. Also, we compare the pdf of d for DNS, LES data of stratocumulus-top boundary layer and POST measurements (Jen-La Plante *et al.*, 2016). By randomly selecting d from its pdf, we perform a 2D a priori test and compare statistics of the constructed velocity increments with DNS velocity increments.

FRACTAL INTERPOLATION TECHNIQUES

Basics

The fractal interpolation technique is an iterative affine mapping procedure to construct the synthetic (unknown) small-scales eddies of the velocity field $\mathbf{u}(\mathbf{x},t)$ from the knowledge of its filtered or coarse-grained field $\tilde{\mathbf{u}}(\mathbf{x},t)$. For the case of three interpolating points $\{(x_i, \tilde{u}_i), i = 0,1,2\}$, the fractal interpolation iterative function system (IFS) is of the form $\{R^2; w_j, j = 1,2\}$, where:

$$w_j \begin{pmatrix} x \\ u \end{pmatrix} = \begin{pmatrix} a_j & 0 \\ c_j & d_j \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{pmatrix} e_j \\ f_j \end{pmatrix}, \quad j = 1,2 \qquad (1)$$

with constraints

$$w_j \begin{pmatrix} x_0 \\ \tilde{u_0} \end{pmatrix} = \begin{pmatrix} x_{j-1} \\ \tilde{u}_{j-1} \end{pmatrix} \quad and \quad w_j \begin{pmatrix} x_2 \\ \tilde{u}_2 \end{pmatrix} = \begin{pmatrix} x_j \\ \tilde{u}_j \end{pmatrix}, \quad j = 1,2 \qquad (2)$$

The parameters a_j, c_j, e_j and f_j can be determined in terms of d_j (called the stretching parameter). The two stretching parameters determine the characteristics of the reconstructed signal and are constrained to be real and lay in the interval (-1,1). Its value is independent of the interpolation points. More details can be found in (Barnley, 1993; Scotti & Meneveau, 1999). Details on stretching parameter estimation using the geometric method of Mazel & Hayes (1992) can be found in Akinlabi *et al.* (2018).

RESULTS

Direct numerical simulation (DNS) data of stratocumulus Cloud-Top for DYCOMS-II RF01 (Mellado, 2017) (courtesy of Prof. J.-P. Mellado from the Max Planck Institute of Meteorology) is used to calculate the stretching parameters spatial probability distribution, using the geometric approach. Details of the simulation is outside the scope of this study and readers are referred to Mellado (2017) for more details. Horizontal profile at a height corresponding to in-cloud region with highest turbulent intensity is used for the calculation of d. First, we investigate the variability of d if the DNS velocity signal is filtered to wavenumber within the inertial range. The signal is filtered with a finite impulse response (FIR) filter of order 30 (a form of a low pass filter) designed using the Hamming window method and we calculate the local estimate of d with the geometric approach explained in Akinlabi *et al.* (2018). The decimation factor is set to equals twice the grid size of the previous iteration step. This was done with decimation function in MATLAB software. Stretching parameter values outside the interval $(-1,1)$ were neglected and absolute value of d was used to calculate the spatial probability distribution function. We applied the same method and filtering technique to LES of stratocumulus Cloud-Top for DYCOMS-II RF01 and Physics of Stratocumulus Top (POST) experimental data. Figure 1 shows the pdf of d for DNS, LES and POST data. We observe some similarity in their pdf except that LES and POST have shorter data for averaging compared to DNS. Figure 2 shows the energy spectra of DNS, filtered and FIT velocity signals. Figure 3 shows the energy spectra of POST experimental data, filtered and FIT velocity signals. All the energy spectra exhibit periodic modulation and this is due to the dyadic nature of the fractal interpolation technique (see figure 7 and 8 of Scotti & Meneveau (1999)). Basu *et al.* (2004) avoid this periodic modulation by applying the discrete Haar wavelet transform. Since the purpose of this work is to obtain statistics of velocity component of atmospheric turbulence, we do not follow this approach and account for the Fourier spectra only. Figure 4 shows the normalized pdfs of DNS, filtered and FIT velocity increments ($\delta u = u(\mathbf{x}+\mathbf{r}) - u(\mathbf{x})$) for $|\mathbf{r}| = 2\eta$, where η is the Kolmogorov's scale, while figure 5 shows the normalized pdfs of POST data, filtered and FIT velocity increments for $|\mathbf{r}| = 2.75m$ which is in the inertial range. The reconstructed velocity signals shows a good fit with both DNS and POST experimental data.

As a next step, we plan to use this model to solve numerically the motion of heavy particles in atmospheric turbulence. The interaction between gravity acceleration and particle inertia has non-trivial effects on the trajectories of particles and their spatial distribution. Also, we aim to improve predictions of averaged settling velocity in LES as a function of particle inertia and the number of iteration used for fractal reconstruction.

CONCLUSION

In this work, we present a fractal subgrid scale model for large eddy simulation of atmospheric flows. With fractal interpolation technique, we construct subgrid velocity field from the knowledge of its filtered or LES grid proposed by Scotti & Meneveau (1999). The (free) stretching parameter determines the characteristics of the reconstructed signal which can be derived from the fractal dimension. In previous literature, the stretching parameter is chosen to be constant in time and space. To account for its spatial variability, we estimate the spatial pdf of the stretching parameter from DNS data of stratocumulus-top boundary layer, using the geometric method proposed by Mazel & Hayes (1992). We observe similarity in the pdf of d when the velocity fields are filtered to wave-numbers within the inertial range. Also, we compare the pdf of d for DNS, LES data of stratocumulus-top boundary layer and Physics of Stratocumulus Top (POST) measurements (Jen-La Plante *et al.*, 2016). By randomly selecting d from its pdf, we perform a 2D a priori test and compare statistics of the constructed velocity increments with DNS velocity increments.

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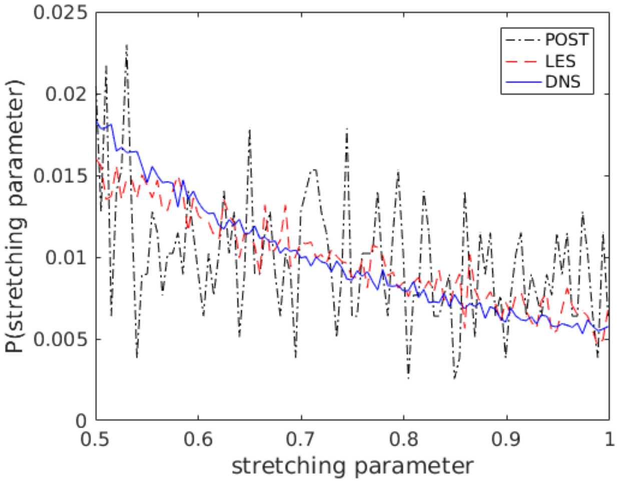

Figure 1. Pdf of stretching parameter d within the interval [0.5, 1] estimated from 2D DNS velocity signal, 2D LES velocity signal and POST experimental data

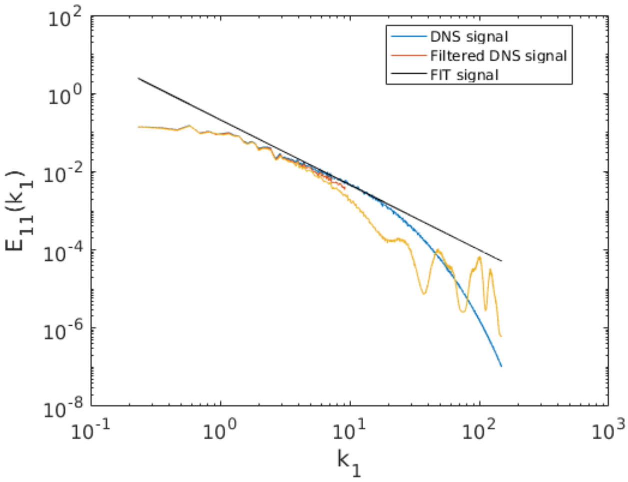

Figure 2. Energy spectra showing $-5/3$ scope for 2D DNS velocity signal

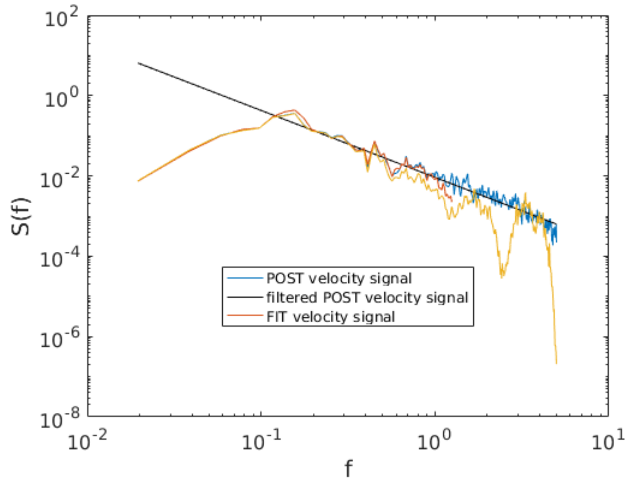

Figure 3. Energy spectra showing $-5/3$ scope for POST experimental data

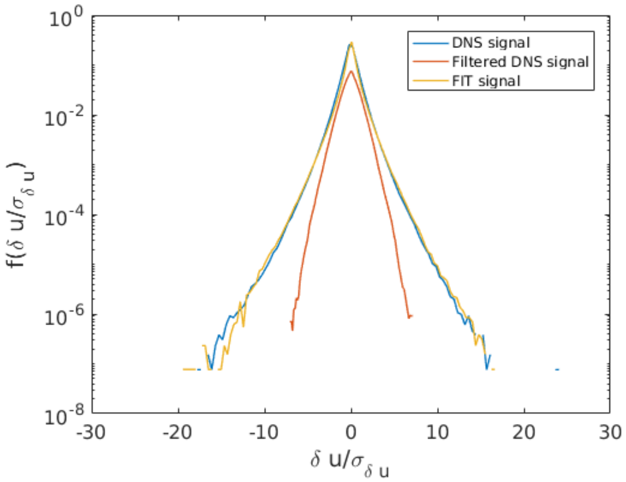

Figure 4. Normalized Pdfs of DNS, filtered and FIT velocity increments (δu) signal

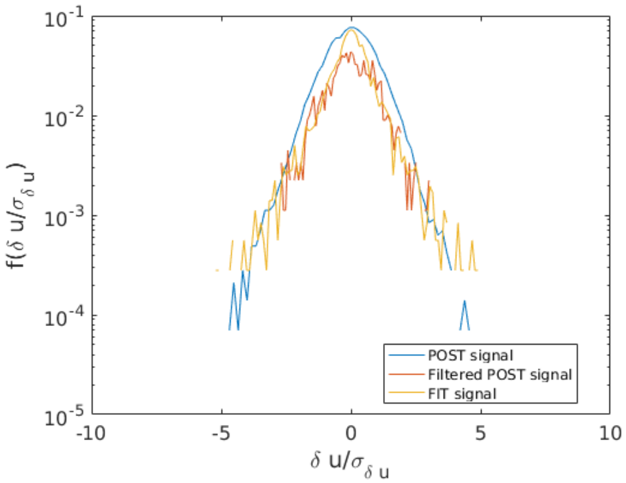

Figure 5. Normalized Pdfs of POST data, filtered and FIT velocity increments (δu) signal