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Fractal reconstruction of sub-grid scales for large eddy simulation of atmospheric turbulence

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Abstract. We present a fractal sub-grid scale model for large eddy simulation (LES) of atmospheric flows. The fractal model is based on the fractality assumption of turbulent velocity field with a dynamical hypothesis based on energy dissipation. The fractal model reconstruct sub-grid velocity field from the knowledge of its filtered values on LES grid, by means of fractal interpolation, proposed by Scotti and Meneveau (1999). The characteristics of the reconstructed signal depends on the (free) stretching parameters, which is related to the fractal dimension of the signal. In previous studies, the stretching parameters was assumed to be constant in space and time and are obtained from experimental velocity signals of homogeneous and isotropic turbulence. To improve this method and account for the stretching parameter variability, we calculate the probability distribution function of the stretching parameter from direct numerical simulation (DNS) data of stratocumulus-top boundary layer (STBL) (courtesy of Prof. J.-P. Mellado from the Max Planck Institute of Meteorology) using the geometric method proposed by Mazel and Hayes. We perform 1D *a priori* test and compare statistics of the constructed velocity increment with DNS velocity increments.

1. Introduction

Atmospheric flows in nature display complex spatial and temporal structures over a wide range of scales from the large synoptic scales $\mathcal{O}(1000km)$ to the smallest dissipative scales $\mathcal{O}(1cm - 1mm)$ with Reynold number $L/\eta \approx \mathcal{O}(10^9)$. All scales play important role in weather prediction including the small scales, which influence droplet collision rate in clouds and affect average settling velocity of droplets with Stokes number much smaller than 1. Direct numerical simulation is an ideal approach to resolve all these scales but it imposes an unrealistic computational cost. Alternatively, large-eddy simulation (LES) allows for significantly improved accuracy in simulating atmospheric (turbulent) flows, by calculating the large scale features of the flow while the interactions between large (resolved) scales and small (unresolved) scales are accounted for by a subgrid-scale model. This type of subgrid models are called functional models. Several functional model has been proposed since the development of the first by Smagorinsky [7] but none has achieve the necessary accuracy for modeling turbulence.

However, structural sub-grid models, an alternative to the functional models, aim at mimicking (some of) the subgrid scales. These models allows for the approximate reconstruction of two-point particle statistics at the subgrid scale, such as relative small scale velocity and particle segregation patterns [12]. Example of structural models are the fractal interpolation [1] and the kinematic simulations based on Fourier modes [8, 11]. Pozorski and Rosa (2018) [11] studied the effect of filtering of the carrier fluid velocity on particle dynamics in forced isotropic

turbulence. They also use kinematic simulations (KS) based on Fourier modes, as a structural subgrid model, to construct the (unresolved) small scales for particle dispersion. They concluded that the kinematic simulation could not mimic the sweeping effect of small scales by the resolved eddies.

Fractal interpolation technique (FIT) was introduced to construct synthetic, fractal subgrid-scale fields applied to large eddy simulation of both steady and freely isotropic decaying turbulence [4]. A (free) parameter used to determine the characteristics of the reconstructed (FIT) signal is called the stretching parameter. In [4], it was assumed that the stretching parameters are constant in space and time in homogeneous and isotropic turbulence. These parameters were set to $d = \pm 2^{-1/3}$. Salvetti et al (2006) [6] use the fractal interpolation technique to reconstruct the velocity field from the filtered values on a coarse (resolved) grid to track Lagrangian particles in turbulent channel flows. They concluded that the constant values of the stretching parameters are not well suited for Lagrangian tracking of particles in near-wall turbulence. Basu et al. (2004) [2] proposed an extension of this work by developing a multiaffine fractal interpolation scheme with stretching parameters $d = -0.887$ and $d = -0.676$ and showed that it preserves the higher-order structure functions and the non-Gaussian probability density function of the velocity increments. They performed an extensive *a priori* analyses of atmospheric boundary layer measurements and argued that the multiaffine closure model should give satisfactory performance in large eddy simulations.

Atmospheric turbulence is known to be inhomogeneous and possesses both internal and external intermittency. Internal intermittency means that large velocity gradients are present at small scales and the probability density function (pdf) of velocity differences at small scales are highly non-gaussian. External intermittency refers to the co-existence of both laminar and turbulent regions in the flow. These attributes make it difficult to synthesize turbulent velocity field using fractal interpolation. The approach of assuming a constant stretching parameter to construct subgrid velocity signals are unrealistic (as reported by [6]) since the local stretching parameters are likely to change randomly in space and time.

The aim of this work is to develop an improvement to the fractal interpolation technique (FIT), which can be used as a closure model for Lagrangian tracking of particles in atmospheric turbulence. To account for the variability of the stretching parameter, we compute the local stretching parameter with a method proposed by Mazel and Hayes (1992) [5] and use its pdf for the fractal interpolation. Also, we compare the statistics of the velocity increments using both the previous version of FIT, the new one and multiaffine fractal interpolation scheme with the statistics of the DNS velocity increments.

2. Fractal interpolation techniques

2.1. Basics

The fractal interpolation technique is an iterative affine mapping procedure to construct the synthetic (unknown) small-scales eddies of the velocity field $\mathbf{u}(\mathbf{x}, t)$ from the knowledge of its filtered or coarse-grained field $\tilde{\mathbf{u}}(\mathbf{x}, t)$. For the case of three interpolating points $\{(x_i, \tilde{u}_i), i = 0, 1, 2\}$, the fractal interpolation iterative function system (IFS) is of the form $\{R^2; w_j, j = 1, 2\}$, where:

$$w_j \begin{pmatrix} x \\ u \end{pmatrix} = \begin{bmatrix} a_j & 0 \\ c_j & d_j \end{bmatrix} \begin{pmatrix} x \\ u \end{pmatrix} + \begin{pmatrix} e_j \\ f_j \end{pmatrix}, \quad j = 1, 2 \tag{1}$$

with constraints

$$w_j \begin{pmatrix} x_0 \\ \tilde{u}_0 \end{pmatrix} = \begin{pmatrix} x_{j-1} \\ \tilde{u}_{j-1} \end{pmatrix} \quad \text{and} \quad w_j \begin{pmatrix} x_2 \\ \tilde{u}_2 \end{pmatrix} = \begin{pmatrix} x_j \\ \tilde{u}_j \end{pmatrix}, \quad j = 1, 2 \tag{2}$$

The parameters a_j, c_j, e_j and f_j can be determined in terms of d_j (called the stretching parameter). The two stretching parameters determine the characteristics of the reconstructed

signal and are constrained to be real and lie in the interval (-1,1). Its value is independent of the interpolation points. Thus, once the stretching parameter is chosen, the remaining parameters a_j, c_j, e_j and f_j are given as

$$a_j = \frac{x_j - x_{j-1}}{x_2 - x_0} \tag{3}$$

$$e_j = \frac{x_2 x_j - x_0 x_j}{x_2 - x_0} \tag{4}$$

$$c_j = \frac{\tilde{u}_j - \tilde{u}_{j-1}}{x_2 - x_0} - d_j \frac{\tilde{u}_2 - \tilde{u}_0}{x_2 - x_0} \tag{5}$$

$$f_j = \frac{x_2 \tilde{u}_{j-1} - x_0 \tilde{u}_j}{x_2 - x_0} - d_j \frac{x_2 \tilde{u}_0 - x_0 \tilde{u}_2}{x_2 - x_0} \tag{6}$$
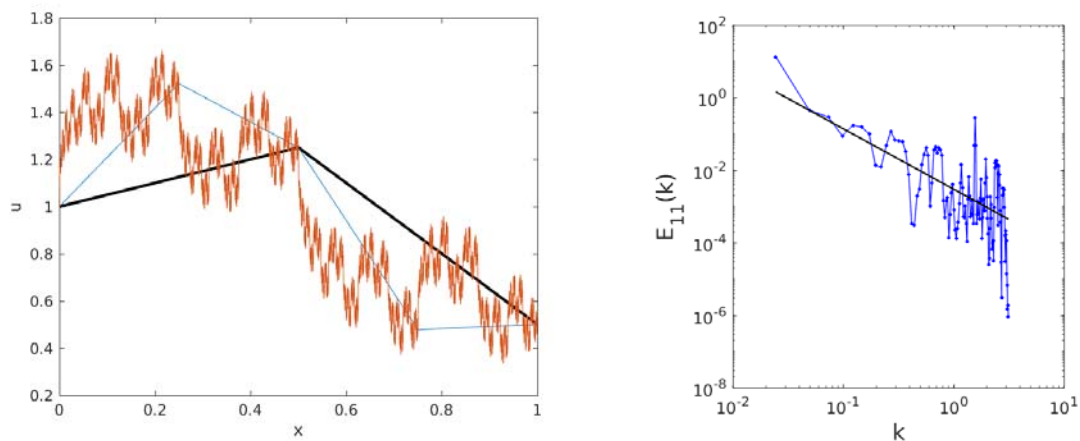

Figure 1. a) Different stages during the construction of a fractal function which interpolates between $(x_i, \bar{u}_i) = (0, 1)$ to $(x_i, \bar{u}_i) = (1, 0.5)$ after 0,1 and 10 iterations with stretching parameter $d = \pm 2^{-1/3}$ b) Energy spectrum of the constructed signal showing $-5/3$ slope.

More details can be found in [1]. To generate a synthetic small scale field, this mapping is simply iterated many more times as shown in figure 1. It was shown in [4] that the stretching parameter depends on the fractal dimension of the signal (scaling exponent spectrum) D as:

$$D = 1 + log_N \sum_{n=1}^{N} |d_n| \tag{7}$$

where N = the number of anchor points $- 1$. The attractor of the above IFS, G, is the graph of a continuous function $u : [x_0, x_2] \to R$, which interpolates the data points $(x_i, \tilde{u}_i)$, if the stretching parameter d_j obey $0 \le |d_j| < 1$. Also, if

$$\sum_{j=1}^{N} |d_j| > 1 \tag{8}$$

and $(x_i, \tilde{u}_i)$, are not collinear, then the fractal dimension of G is the unique real solution D of

$$\sum_{j=1}^{N} |d_j| a_j^{D-1} = 1 \tag{9}$$

2.2. Stretching parameter estimation

As discussed in [5], the local stretching parameter d can be calculated using an analytic and geometric approach. Both method gives approximate results as shown in figure 3-6 of [5]. For this study, we use the geometric approach, which can be understood as a reverse process of fractal interpolation. For example, for 5 interpolation points $\{(x_i, u_i), i = 0, 1, 2, 3, 4\}$, let μ be the vertical distance between the middle interpolation point (x_2, u_2) and a straight line between the end points (x_0, u_0) and (x_4, u_4). The value of μ is positive if the interpolation points are above the straight line and negative otherwise. Let ν_1 be the vertical distance between the first three interpolation points $\{(x_i, u_i), i = 0, 1, 2\}$ and a straight line between (x_0, u_0) and (x_2, u_2) while ν_2 be the vertical distance between the last three interpolation points $\{(x_i, u_i), i = 2, 3, 4\}$ and a straight line between (x_2, u_2) and (x_4, u_4). Both ν_1 and ν_2 are positive if their respective interpolation points are above their respective straight lines and negative otherwise. Then the stretching parameters d_1 and d_2 are ν_1/μ and ν_2/μ respectively. An illustration of this calculation is presented in figure 2.
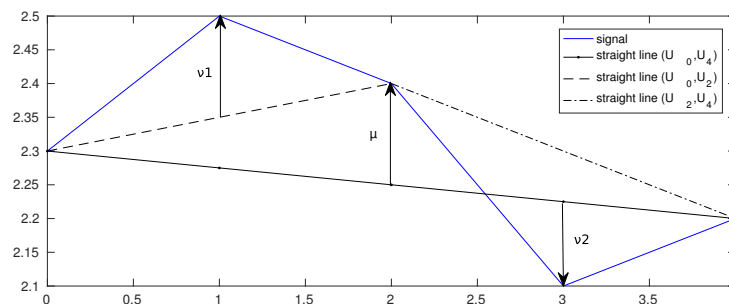

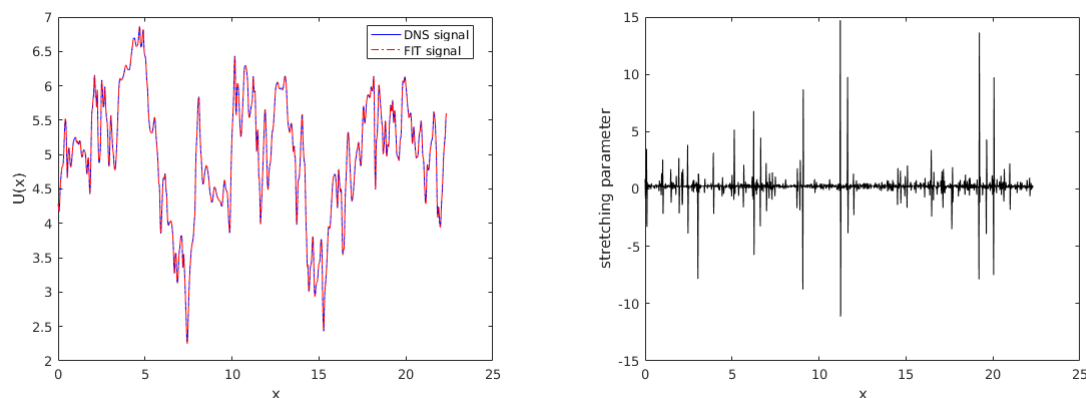**Figure 2.** Geometric approach for stretching parameter calculation.

Figure 3. a) 1D DNS velocity signals showing the original and the FIT reconstructed signal b) The stretching parameters used to construct FIT signal in figure (a).

Figure 3a shows the 1D original and reconstructed FIT signal with calculated d values. If the procedure for d retrieval is correct both the original and the FIT signal should be identical, which is the case as observed in figure 3a. The stretching parameter as shown in figure 3b varies significantly in space and has values outside the interval $(-1, 1)$ because the original signal is highly intermittent. Regions of high velocity gradients in intermittent signals can not be

reproduced with FIT since d should only take values within the interval $(-1, 1)$. If we place a constraint, such that $|d| \leq 1$, only some of the sub-grid scales will be reproduced. We compare this geometric approach of calculating the stretching parameter with constant values proposed by [4] and [2] in section 3.2.

3. Results
Direct numerical simulation (DNS) data of stratocumulus Cloud-Top for DYCOMS-II RF01 [3] (courtesy of Prof. J.-P. Mellado from the Max Planck Institute of Meteorology) is used to calculate the stretching parameters spatial probability distribution, using the geometric approach. Details of the simulation is outside the scope of this study and readers are referred to [3] for more details.

3.1. Stretching parameter spatial probability distribution function
Horizontal profile at a height corresponding to in-cloud region with highest turbulent intensity is used for the calculation of d. First, we investigate the variability of d as the DNS velocity signal is filtered, starting with the fully resolved DNS signal and filtering successively down to wavenumber from the inertial range. In each iteration step, the signal is filtered with a finite impulse response (FIR) filter of order 30 (a form of a low pass filter) designed using the Hamming window method and we calculate the local estimate of d with the geometric approach explained in section 2.2. The decimation factor is set to equals twice the grid size of the previous iteration step. This was done with decimation function in MATLAB$^{\circledR}$ software. Stretching parameter values outside the interval $(-1, 1)$ were neglected and absolute value of d was used to calculate the spatial probability distribution function.
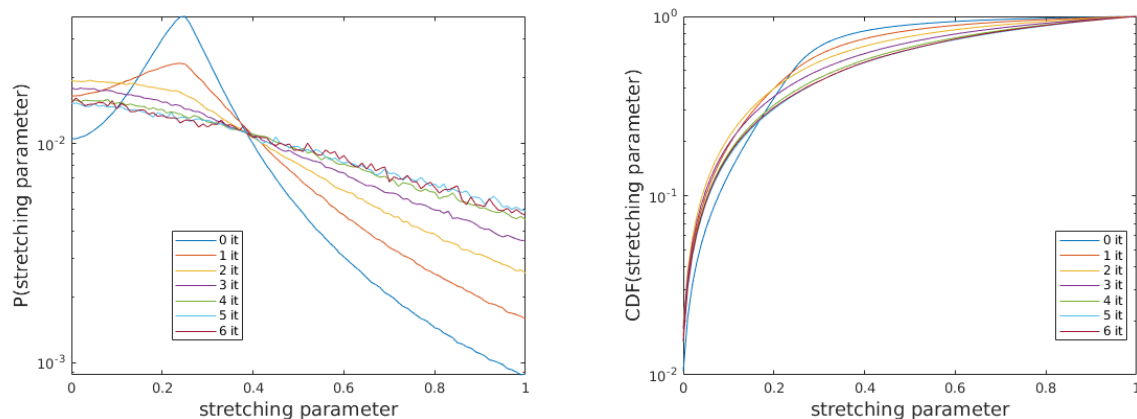

Figure 4. a) Spatial probability distribution of stretching parameter for $1D$ DNS velocity signals at different iteration step b) Its cumulative probability distribution at different iteration step

Figure 4 shows the spatial probability distribution of stretching parameter for 1D DNS velocity signals for different iteration step and their cumulative probability distribution function. In figure 4a, the pdfs change significantly in the first four iterations but seems to be self-similar when filtered with wavenumbers in the inertial range. Figure 5 shows the average stretching parameter and fractal dimension of $1D$ DNS velocity signal for different iteration steps. In figure 5a, the average stretching parameter are seen to be smaller than 0.5 as compared to figure 2 of [6] where an average value of 0.6 and 0.7 were reported for x and y directions, respectively. Our result with somewhat smaller d might be due to the presence of external intermittency (laminar
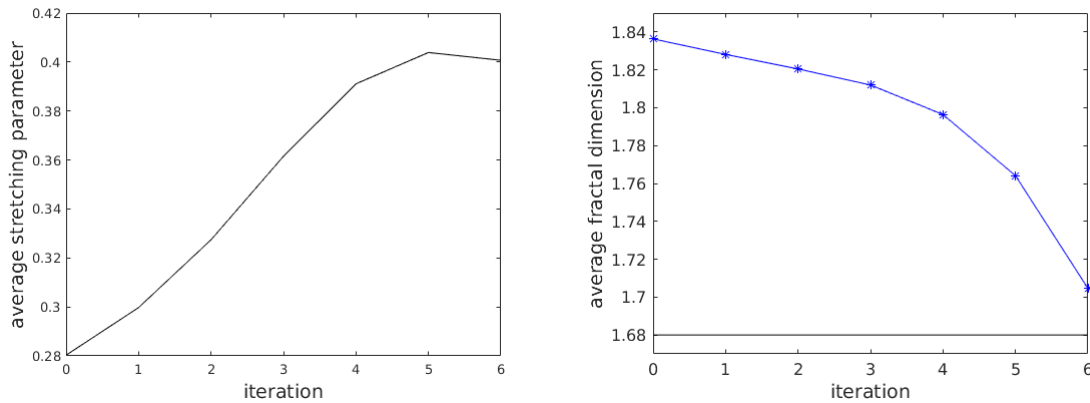
Figure 5. a) Average stretching parameter of $1D$ DNS velocity signals for different iteration step b) Its average fractal dimension showing the 5/3 scaling.
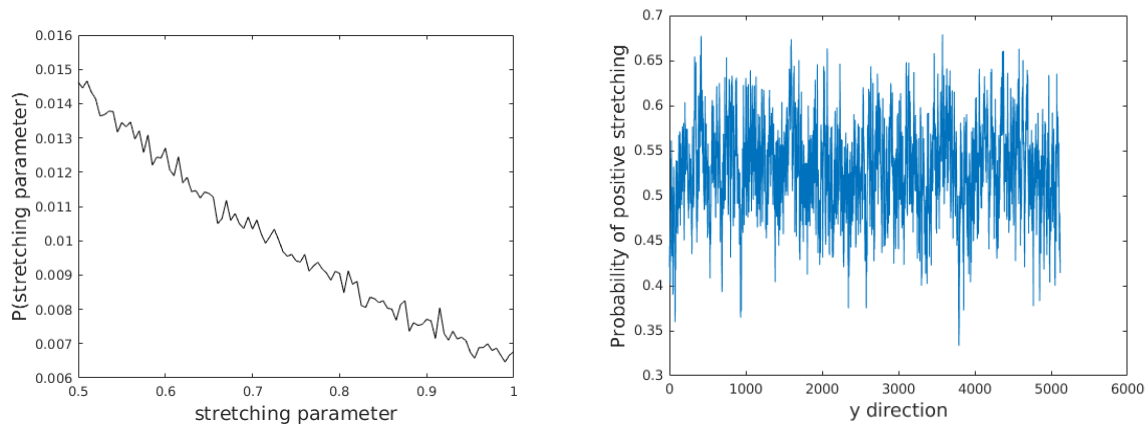

Figure 6. a) Pdf of stretching parameter d within the interval [0.5, 1] estimated from 1D DNS velocity signal (b) Probability of positive stretching parameter

regions will give zero or close to zero stretching parameter). If we ignore $|d| < 0.5$, we will obtain similar average stretching parameter as in [6] (see fig. 6a). Still, with our current results the average fractal dimension decreases to 5/3 scaling, as expected in the inertial range, see figure 5b.

Since this work focuses on constructing inertial range subgrid scales of atmospheric turbulence, we focus on the pdf of d when filtered with wavenumbers from the inertial range. The pdf of d, as shown in figure 6a, will be used to construct sub-grid scale for the filtered DNS velocity signal in section 3.2. We also investigate the probability of having positive (or negative) stretching parameter and figure 6b shows that, on the average, there exist an equal probability of having positive and negative stretching parameters.

3.2. 1D fractal interpolation of DNS of stratocumulus cloud-top for DYCOMS-II RF01
We filter each 1D DNS velocity signal of stratocumulus cloud-top in the y direction with inertial range wavenumber and use the pdf of d in figure 6a to construct subfilter scale using fractal interpolation technique. We present the spectra and pdfs of the velocity increments of DNS, filtered and FIT reconstructed signal. We also present the spectra and pdfs of the velocity

increments of FIT reconstructed signal using a constant values of d as previously proposed.
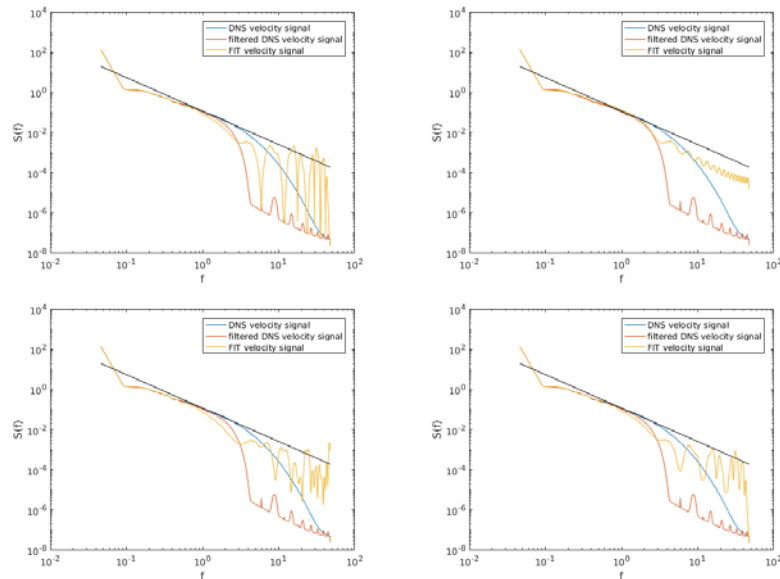

Figure 7. Energy spectra showing $-5/3$ scope (a) with constant stretching parameter $d = \pm 2^{-1/3}$ (b) with constant stretching parameters $d = 0$ and $d = 0.93$ (c) with constant stretching parameters $d = -0.887$ and $d = -0.676$ (d) with stretching parameters from section 3.1

Figure 8. Normalized Pdfs of DNS, filtered and FIT velocity increments (δu) signal (a) with constant stretching parameter $d = \pm 2^{-1/3}$ (b) with constant stretching parameters $d = 0$ and $d = 0.93$ (c) with constant stretching parameters $d = -0.887$ and $d = -0.676$ (d) with stretching parameters from section 3.1
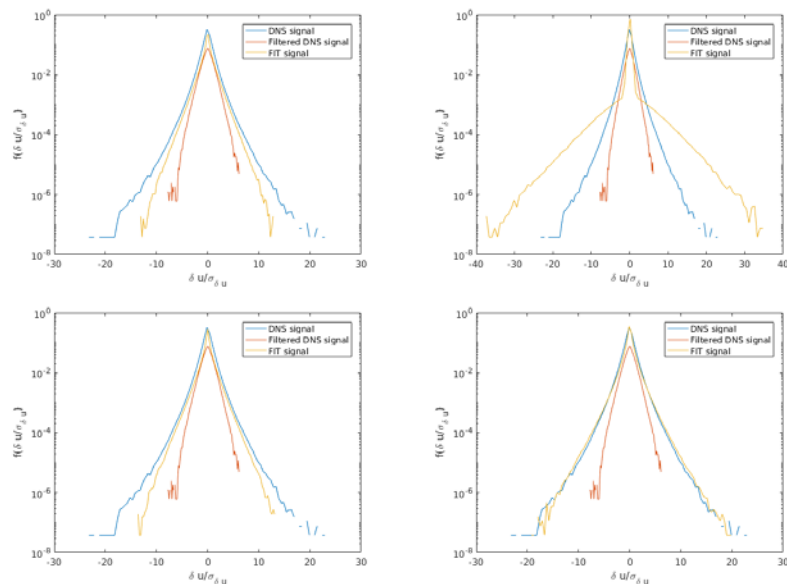
Here, we focus on extending the inertial range scales and not explicit recovery of the

dissipative range of the spectra. Figure 7 shows the energy spectra of DNS, filtered and FIT velocity signals. All the energy spectra exhibit periodic modulation and this is due to the dyadic nature of the fractal interpolation technique (see figure 7 and 8 of [4]). Basu et al. (2004) [2] avoid this periodic modulation by applying the discrete Haar wavelet transform. Since the purpose of this work is to obtain statistics of velocity component of atmospheric turbulence, we do not follow this approach and account for the Fourier spectra only. Figure 8 shows the normalized pdfs of velocity increment (δu) for DNS, filtered and FIT velocity signals. The best fit with DNS data is observed for the reconstruction with the new pdf of stretching parameter, shown in figure 8d.

4. Conclusion

In this work, we present a fractal subgrid scale model for large eddy simulation of atmospheric flows. With fractal interpolation technique, we construct subgrid velocity field from the knowledge of its filtered or LES grid proposed by [4]. The (free) stretching parameter determines the characteristics of the reconstructed signal which can be derived from the fractal dimension. In previous literature, the stretching parameter is chosen to be constant in time and space. To account for its spatial variability, we estimate the spatial pdf of the stretching parameter from DNS data of stratocumulus-top boundary layer, using the geometric method proposed by [5]. We compare constant estimate of the stretching parameter with spatial pdf of the stretching parameter to construct sub-grid velocity. We present the energy spectra and pdfs of the normalized velocity increments.

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